# Scheme of Work 

## Cambridge IGCSE ${ }^{\oplus}$ / Cambridge IGCSE $^{\oplus}$ (9-1) Mathematics 0580 / 0980

For examination from 2020


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Contents
Introduction .....  4
1 Number ..... 7
2 Algebra and graphs ..... 20
3 Coordinate geometry ..... 33
4 Geometry ..... 36
5 Mensuration ..... 41
6 Trigonometry. ..... 44
7 Vectors and transformations ..... 48
8 Probability ..... 50
9 Statistics ..... 53

## Introduction

This scheme of work has been designed to support you in your teaching and lesson planning. Making full use of this scheme of work will help you to improve both your teaching and your learners' potential. It is important to have a scheme of work to ensure that the syllabus is fully covered. You can choose what approach to take as you know the nature of your institution and the abilities of your learners; what follows is just one possible approach. Always check the syllabus for the content of your course.

Suggestions for independent study (I) and formative assessment (F) are also included. Opportunities for differentiation are indicated as Extension activities; there is the potential for differentiation by resource, grouping, expected level of outcome, and degree of support by teacher, throughout the scheme of work. Timings for activities and feedback are left to the judgment of the teacher, according to the level of the learners and size of the class. Length of time allocated to a task is another possible area for differentiation.

## Guided learning hours

Guided learning hours give an indication of the amount of contact time you need to have with your learners to deliver a course. Our syllabuses are designed around 130 hours for Cambridge IGCSE courses. The number of hours may vary depending on local practice and your learners' previous experience of the subject. The table below gives some guidance about how many hours we recommend you spend on each topic area.

| Topic | Suggested teaching time (hours / \% of the course) | Suggested teaching order |
| :---: | :---: | :---: |
| 1 Number | Core: 39-46 hours (30-35\%); Extended: 19-26 hours (15-20\%) | 1 |
| 2 Algebra and Graphs | Core: 19-26 hours; (15-20\%); Extended 39-46 hours (30-35\%) | 3 |
| 3 Geometry | Core: 10 hours (8\%) | 4 |
| 4 Co-ordinate Geometry | 6-9 hours (5-7\%) | 2 |
| 5 Mensuration | 10 hours (8\%) | 5 |
| 6 Trigonometry | 6-9 hours (5-7\%) | 6 |
| 7 Vectors and Transformations | 6-9 hours (5-7\%) | 7 |
| 8 Probability | 9 hours (7\%) | 8 |
| 9 Statistics | 10 hours (8\%) | 9 |

## Resources

The up-to-date resource list for this syllabus, including textbooks endorsed by Cambridge International, is listed at www.cambridgeinternational.org
Endorsed textbooks have been written to be closely aligned to the syllabus they support and have been through a detailed quality assurance process. As such, all textbooks endorsed by Cambridge International for this syllabus are the ideal resource to be used alongside this scheme of work as they cover each learning objective. Past papers and Example Candidate Responses for syllabus 0580 are relevant and applicable to syllabus 0980 , so we suggest you visit the webpage for 0580 on our School Support Hub for further teaching materials.

## School Support Hub

The School Support Hub www.cambridgeinternational.org/support is a secure online resource bank and community forum for Cambridge teachers, where you can download specimen and past question papers, mark schemes and other resources for 0580 (which are also relevant for 0980). We also offer online and face-to-face training; details of forthcoming training opportunities are posted online. This scheme of work is available as PDF and an editable version in Microsoft Word format; both are available on the School Support Hub at www.cambridgeinternational.org/support. If you are unable to use Microsoft Word you can download Open Office free of charge from www.openoffice.org

## Resource Plus

Throughout this scheme of work, you will find references to resources on the Resource Plus platform.

Resource Plus
Skills Pack: Venn diagrams

Resource Plus is a premium digital teaching and learning resource for Cambridge syllabuses that supports teachers and learners around the world in the most challenging topics and skills. The topics of focus have been defined by quantitative analysis of exam results, as well as qualitative scrutiny of Principal Examiner's reports, engagement with Cambridge teachers, and review of the latest literature and research. Resource Plus for Cambridge IGCSE Mathematics 0580 includes: Skills Packs containing detailed lesson plans, worksheets and answers; PowerPoint ${ }^{\oplus}$ presentations that can be used in lessons, containing class activities and notes to guide teaching; Demonstration videos designed to explain difficult concepts to learners; Additional teaching resources including onscreen mathematical models and templates; and Past paper questions focused on challenging topics.

Resource Plus also offers a wide range of other materials for you to use in your classroom. To try a demo, find out more, or to subscribe, visit www.cambridgeinternational.org/resourceplus

## Websites

This scheme of work includes website links. Cambridge Assessment International Education is not responsible for the accuracy or content of information contained in these sites. The inclusion of a link to an external website should not be understood to be an endorsement of that website or the site's owners (or their products/services). The website pages referenced in this scheme of work were selected when the scheme of work was produced. Other aspects of the sites were not checked and only the particular resources are recommended.

## Command words

The syllabus now includes a list of command words used in the assessment. You should introduce your learners to these words as soon as possible and use them throughout your teaching to ensure that learners are familiar with them and their meaning. See the syllabus for more detail.

How to get the most out of this scheme of work - integrating syllabus content, skills and teaching strategies
We have written this scheme of work for the Cambridge IGCSE Mathematics 0580 and Cambridge IGCSE Mathematics (9-1) 0980 syllabuses and it provides some ideas and suggestions of how to cover the content of the syllabus. We have designed the following features to help guide you through your course.


## 1 Number

| Syllabus <br> ref. | Learning objectives |
| :--- | :--- |
| C1.1 and <br> E1.1 | Identify and use natural numbers, <br> integers (positive, negative and zero), <br> prime numbers, square numbers, <br> common factors and common <br> multiples, rational and irrational <br> numbers (e.g. $\pi, \sqrt{2}$ ), real numbers, <br> reciprocals <br> Includes expressing numbers as a <br> product of prime factors. <br> Finding the Lowest Common <br> Multiple (LCM) and Highest <br> Common Factor (HCF) of two <br> numbers. |

A useful starting point would be to revise positive and negative numbers using a number line and explain the difference between natural numbers and integers. Learners would find it useful to have a definition of the listed terms (e.g. factor, multiple, square number) which can be found on the Maths Revision website (www.mathsrevision.net). If the link doesn't work: from the home page, select 'GCSE > number > numbers'. (I)

A fun activity is to allocate a number to each learner in the class and ask them to stand up if their number exhibits a property that you call out. For example, you might call out "a multiple of 4"; "a factor of 18 ", etc. Use this to show interesting facts, such as prime numbers will have 2 people standing up (this emphasises that 1 is not prime); and square numbers will have an odd number of people standing up. You can use this activity to highlight common factors/common multiples for pairs of numbers. This could be extended to HCF and LCM.

A follow-on activity would be for learners to identify a number from a description of its properties. For example, say to the class "which number less than 50 has 3 and 5 as factors and is a multiple of 9 ?" You could also ask learners to make up their own descriptions and test one another.

Another interesting task is to look a Fermat's discovery, which shows that some prime numbers are the sum of two squares, e.g. $29=25+4=5^{2}+2^{2}$. Learners could see what other prime numbers they can form in this way, and any they can't form in this way. Learners can look for a rule that tests if a prime can be made like this. (I)

Look at how to write any integer as a product of prime numbers. One method that can be used is the factor tree approach which can be found online. Demonstrate this technique to your learners, then ask them to practise using the method to write other numbers as products of primes. Then ask learners to look at finding the product of primes of other numbers, for example $60,450,42,315$, but this time they should be encouraged to look for alternative methods, for example by researching on the internet; another useful method is the repeated division method. (I)

Give learners a definition of the terms rational, irrational and real numbers, which you can find on the Maths is Fun website (www.mathsisfun.com). If the link doesn't work, from the home page: click on 'Index $>10$ upwards $>$ numbers $>$ Irrational Numbers. The website also includes questions on rational and irrational numbers for learners to try. These start simple and soon become more challenging. (I) (F)

| Syllabus ref. | Learning objectives | Suggested teaching activities |
| :---: | :---: | :---: |
| C1.2 | Understand notation of Venn diagrams. <br> Definition of sets <br> e.g. <br> $A=\{x: x$ is a natural number $\}$ <br> $B=\{a, b, c, \ldots \ldots$. | It is useful to start by introducing/revising simple Venn diagrams. For example, group people who wear glasses in one circle and people with brown hair in another circle, and ask learners to describe the people in the overlapping region. You can encourage learners to actively participate by asking them to place physical objects into the regions of a Venn diagram. You could even get learners to create their own version of the Venn diagram by moving around the classroom based on their appearance; for example, as per the glasses and hair colour example above. You could introduce the idea of a union in the intersection visually, and then link this to the notation that they need to use. (I) <br> There is some useful material on sets and set notation on the Maths Is Fun website (www.mathsisfun.com). Search for 'Introduction to Sets' and 'Sets and Venn Diagrams' These resources also include some multiple choice questions that learners could use to check their understanding. (F) <br> Venn diagrams are a great way to visualise the structure of set relationships. They can be used to help visualise a broad range of problems across the mathematics curriculum where you want to explore the relationships between groups. For example, they can be used to help solve probability questions (see syllabus ref. C8 .1, C8 .5, E8 .6, below). They can also be used across strands as a way of enhancing conceptual understanding by the use of multiple representations. <br> While Venn diagrams work well for two or even three sets, they very quickly break down when the number of sets gets beyond three. It is important that learners understand that they a tool for visualising a problem but not really the solution to the problem that itself. <br> Resource Plus <br> Skills Pack: Venn diagrams <br> This Skills Pack includes lessons that cover the following: <br> using Venn diagrams <br> formal notation used with Venn diagrams <br> constructing Venn diagrams to solve problems <br> calculating simple probabilities using Venn diagrams. |

Syllabus ref.

E1. 2
Use language, notation and Venn diagrams to describe sets and represent relationships between sets.

## Definition of sets

e.g. $A=\{x: x$ is a natural number $\}$
$B=\{(x, y): y=m x+c\}$
$C=\{x: a$ Y $x$ Y $b\}$
$D=\{a, b, c, \ldots\}$

## Notation

Number of elements in set $A ; \mathrm{n}(A)$
"...is an element of..." $A$;
$\in$
"...is not an element of..."; $\notin$
Complement of set $A$;$A^{\prime}$

The empty set; $\varnothing$
Universal set; $\mathcal{E}$
$A$ is a subset of $B ; \quad A \subseteq B$
$A$ is a proper subset of $B ; \quad A \subset B$
$A$ is not a subset of $B ; \quad A \nsubseteq B$
$A$ is not a proper subset of $B ; A \not \subset B$
Union of $A$ and $B ; \quad A \cup B$
Intersection of $A$ and $B ; \quad A \cap B$

## Suggested teaching activities

It is useful to start with revising simple Venn diagrams. For example, group people who wear glasses in one circle and people with brown hair in another circle, and ask learners to describe the people in the overlapping region.

This can be extended to general Venn diagrams concentrating more on the shading of the regions representing the sets $A \cup B, A \cap B, A^{\prime} \cup B, A \cup B^{\prime}, A^{\prime} \cap B, A \cap B^{\prime}, A^{\prime} \cup B^{\prime}$ and $A^{\prime} \cap B^{\prime}$ helping learners to understand the notation.

Learners would find it useful to know that $(A \cup B)^{\prime}$ is the same as $A^{\prime} \cap B^{\prime}$ and that $(A \cap B)^{\prime}$ is the same as $A^{\prime} \cup B^{\prime}$. Make sure that learners understand the language associated with sets and Venn diagrams.

Learners need to be able to distinguish between a subset and a proper subset.
The work on Venn diagrams can be extended to look at unions and intersections when there are three sets.

## Resource Plus

Skills Pack: Venn diagrams
This Skills Pack includes lessons that cover the following:

- using Venn diagrams
- formal notation used with Venn diagrams
- constructing Venn diagrams to solve problems
- calculating simple probabilities using Venn diagrams.

| Syllabus ref. | Learning objectives | Suggested teaching activities |
| :---: | :---: | :---: |
| C1.3 and E1.3 | Calculate with squares, square roots, cubes and cube roots and other powers and roots of numbers, e.g. work out $3^{2} \times \sqrt[4]{16}$ | Using simple examples, illustrate squares, square roots, cubes and cube roots of integers. <br> Extend this by asking more able learners to find the square and cube of fractions and decimals without using a calculator; you might need to cover topic 1.8 first to help with this. <br> Show how to find the square root of an integer by repeated subtraction of consecutive odd numbers until you reach zero. For example, for 25 subtract in turn $1,3,5,7$, and then 9 to get to 0 . Five odd numbers have been subtracted so the square root of 25 is 5 . Ask learners to investigate this method for other, larger, square numbers. (I) <br> Explain to learners that the square number 121 is palindromic (when the digits are reversed it is the same number). Challenge learners to find all the palindromic square numbers less than 1000. (I) <br> To check their understanding, learners can then try the specimen paper question. (F) |
| C1.4 and E1.4 | Use directed numbers in practical situations, e.g. temperature changes, flood levels | Draw a number line from - 20 to 20. Point to various numbers (both positive and negative) and ask learners questions such as "what is 5 more than this number?"; "What is 6 less than this number?" You can keep it simple by using only integers. Extension activity: extend the task by using decimals or fractions. <br> Look at directed numbers in the context of practical situations such as temperature changes, flood levels, bank credits and debits. Learners can investigate a variety of temperature changes involving positive and negative temperatures, using the statistics for over 29000 cities on weatherbase.com (www.weatherbase.com). (I) To check their understanding, learners can then try the past paper question. (F) |
| C1.5 and E1.5 | Use the language and notation of simple vulgar and decimal fractions and percentages in appropriate contexts. <br> Recognise equivalence and convert between these forms. | Give learners a definition of the relevant terms (e.g. numerator, denominator, equivalent fractions, simplify, vulgar fraction, improper fraction, mixed number, decimal fraction, and percentage). Ask learners to produce a crossword with the terms defined. Ask them to add any other terms that they can think of to do with fractions, decimals and percentages. Crosswords can be easily created using the excellent online software at EclipseCrossword.com (www.eclipsecrossword.com) (I) <br> Use clear examples and questions to cover converting between fractions, decimals and percentages. Learners should understand how to use place value (units, tenths, hundredths, etc.) to change a simple decimal into a fraction. For example <br> 0.3 has 3 in the tenths column so it is $\frac{3}{10}$. |


| Syllabus ref. | Learning objectives | Suggested teaching activities |
| :---: | :---: | :---: |
|  | Includes the conversion of recurring decimals to fractions, e.g. change $0 . \dot{7}$ to a fraction | Look at the online lesson 'converting repeating decimals to fractions' at Basic-Mathematics.com (www.basicmathematics.com) to demonstrate how to convert recurring decimals to fractions. It uses the following method: $\begin{aligned} x & =0.15151515 \ldots . \\ 100 x & =15.15151515 \ldots . \text { subtract these to get } \\ 99 x & =15 \text { so } x=\frac{15}{99}=\frac{5}{33} \end{aligned}$ <br> To check their understanding, learners can then try the past paper question. (F) |
| C1.6 and E1.6 | Order quantities by magnitude and demonstrate familiarity with the symbols $=, \neq,>,<, \geq, \leq$ | A good active learning approach to this topic is to give learners a set of cards with the symbols $=, \neq,>,<, \geq, \leq$. Ask them to choose which card should go in between pairs of quantities that you give them. For example, 400 m and $4000 \mathrm{~cm} ; 20 \%$ and $0.2 ;-8$ and -10 , etc. Extend this by asking learners to consider when, or if, more than one card can be used (e.g. $\neq$ can be used in place of $>$ or $<$ ). <br> Give learners a list of fractions, decimals and percentages. Ask them to order these by magnitude using the inequality signs. <br> To check their understanding, learners can then try the past paper question. (F) |
| C1.7 and <br> E. 17 | Understand the meaning of indices (fractional, negative and zero) and use the rule of indices $5^{\frac{1}{2}}=\sqrt{5}$ <br> Find the value of $5^{-2}, 100^{\frac{1}{2}}, 8^{-\frac{2}{3}}$ Work out $2^{-3} \times 2^{4},\left(2^{3}\right)^{2},\left(2^{-3} \div 2^{4}\right)$ <br> Use the standard form $A \times 10^{n}$ where $n$ is a positive or negative integer, and $1 \leq A<10$. <br> Convert numbers into and out of standard form. | Start with by revising the meaning of positive indices and the basic rules of indices such as $3^{3} \times 3^{5}=3^{8}, 5^{4} \div 5^{3}=5^{1}=5$, etc. Give simple examples to revise writing an integer as a product of prime numbers, including writing answers using index notation. Try to avoid using 2 as the number you work with initially as this can encourage the misconception that $2^{3}$ means $2 \times 3$ because of the exception that is $2^{2}$. This exception can be discussed once the concept is secure. <br> An interesting challenge for learners is the puzzle 'Power Crazy' on the Nrich website (https://nrich.maths.org). Ask learners to work in groups to complete the challenge. This can be extended to 'Excel investigation: Power Crazy' on the same website. This example also illustrates that a spreadsheet can be used as a tool to make calculations, but we need our brains to solve the problem using reasoning. <br> Move on to negative, zero and fractional indices. <br> Useful examples are $2^{-1}=2^{(2-3)}=\frac{2^{2}}{2^{3}}=\frac{1}{2}$ and $2^{0}=2^{(3-3)}=\frac{2^{3}}{2^{3}}=1$ <br> You can move on to introducing fractional indices by relating them to roots (of positive integers), for example $4^{\frac{1}{2}} \times 4^{\frac{1}{2}}=4^{1}=4$ so $4^{\frac{1}{2}}=\sqrt{4}=2$. The rules of indices can be used to show how values such as $16^{\frac{3}{4}}$ can be simplified. <br> Learners should try lots of examples and questions. (I) |


| Syllabus ref. | Learning objectives | Suggested teaching activities |
| :---: | :---: | :---: |
|  | Calculate with values in standard form. | The next step is to introduce standard form. The Maths Is Fun website (www.mathsisfun.com) can be used to make links between the rules of indices and standard notation. Search for 'Index notation and powers of 10'. <br> Learners could explore the f following problem: <br> Using $a=6 \times 10^{3}$ and $b=3 \times 10^{2}$ determine which of these calculations gives the largest solution and which gives the smallest. <br> It is important learners understand that standard form is way of writing very large and very small numbers. For example, it is used on a scientific calculator when a number is too large or too small to fit on the screen. Being able to write numbers in standard form depends on a secure understanding of place value. This understanding is fundamental in manipulating large and small numbers, both mentally and in written form. <br> This video 'Powers of ten and the relative size of things in the universe' available on the Eames office official website (www.eamesoffice.com); it is also available on YouTube (https://www.youtube.com/watch?v=0fKBhvDjuy0). This is very good for helping learners to understand the concept of magnitude. <br> The next step is to give learners a range of examples showing how to write numbers in standard form and vice-versa. Emphasise to learners that different calculators display standard form in different ways and check that they know how to input numbers in standard form into their calculator. <br> Extend this by using the four rules of calculation with numbers in standard form, both with and without a calculator. Emphasise common errors, for example, if learners are asked to work out the answer to $2.4 \times 10^{4}-2 \times 10^{4}$ in standard form it is common to see an answer of $0.4 \times 10^{4}$. Point out that although $2.4 \times 10^{4}-2 \times 10^{4}=0.4 \times 10^{4}$ the answer is not in standard form, since 0.4 is less than 1 . <br> Ask learners to try the 'Standard form worksheet' from the TES website (www.tes.com). |


| Syllabus ref. | Learning objectives | Suggested teaching activities |
| :---: | :---: | :---: |
| C1.8 and E1.8 | Use the four rules for calculations with whole numbers, decimals and fractions (including mixed numbers and improper fractions), including correct ordering of operations and use of brackets. <br> Applies to positive and negative numbers. | A good starting activity is to ask learners to work in groups to use four 4 s and the four rules for calculations to obtain all the whole numbers from 1 to 20 , e.g. $4+4 \times 4-4=16$. <br> The next step is to look at long multiplication, and short and long division. You can see the traditional and repeated subtraction (chunking) examples on the BBC bitesize website (www.bbc.co.uk/schools/gcsebitesize/maths); if the link doesn't work, search for 'Long multiplication and division' on the BBC Bitesize website. This should be revision for most but is worth spending a bit of time on to ensure learners are confident in the methods. <br> Clarify the order of operations, including the use of brackets. Highlight common errors such as working from left to right instead of using the order of operations rule, BIDMAS (Brackets Indices Division Multiplication Addition and Subtraction). Give learners some examples that illustrate the rules for multiplying and dividing with negative numbers. <br> Extend this to using the four rules with fractions (including mixed numbers) and decimals. It is important that learners can do these calculations both with and without the use of a calculator as they may be expected to show working. learners to try the practice questions. (I) |
| C1.9 and E1.9 | Make estimates of numbers, quantities and lengths, give approximations to specified numbers of significant figures and decimal places and round off answers to reasonable accuracy in the context of a given problem. | A simple starting point is to revise rounding numbers to the nearest $10,100,1000$, etc., or to a set number of decimal places. Show learners how to round a number to a given number of significant figures, explaining the difference and similarities between significant figures and decimal places. <br> It is helpful to explain common misconceptions such as 43.98 to 1 d.p. is 44.0 not 44 . Emphasise that for this syllabus, nonexact answers are required to three significant figures unless the question says otherwise. Revision of estimating and rounding can be found on the Math.com website (www.math.com); if the link doesn't work, search for 'Estimating and rounding decimals’. |


| Syllabus ref. | Learning objectives | Suggested teaching activities |
| :---: | :---: | :---: |
| $\begin{aligned} & \text { C1.10 and } \\ & \text { E1.10 } \end{aligned}$ | Give appropriate upper and lower bounds for data given to a specified accuracy, e.g. measured lengths. <br> Obtain appropriate upper and lower bounds to solutions of simple problems given data to a specified accuracy, e.g. the calculation of the perimeter or the area of a rectangle. | Start this topic with examples to determine upper and lower bounds for data. Use simple examples and then progressively harder ones, such as: 'a length, $l$, measured as 3 cm to the nearest millimetre has lower bound 2.95 cm and upper bound 3.05 cm '. Emphasise that the bounds, in this case, are not 2.5 and 3.5, which would be a common misconception. Show learners how this information can be written using inequality signs, e.g. $2.95 \mathrm{~cm} \leq l<3.05 \mathrm{~cm}$. <br> For extended learners, move on to looking at upper and lower bounds for quantities calculated from given formulae. <br> To check their understanding, learners can then try the past paper question. (F) <br> Resource Plus <br> Skills Pack: Accuracy and bounds <br> This Skills Pack includes lessons that cover the following: <br> - accuracy and bounds to the nearest 10,100 or 1000 <br> - ... to 3 decimal places <br> - ... to significant figures <br> - substituting bounds into formulae. |
| C1.11 and E1.11 | Demonstrate an understanding of ratio and proportion. <br> Calculate average speed. <br> Use common measures of rate. <br> To include numerical problems involving direct and inverse proportion. Use ratio and scales in practical situations. Formulae for other rates will be given in the question e.g. pressure and density. | Learners will find it useful to have a definition of ratio with a practical demonstration, for example the ratio of different coloured beads on a necklace. <br> Look at examples illustrating how a quantity can be divided into a number of unequal parts. For instance: ‘share $\$ 360$ in the ratio $2: 3: 7$ '. Move on to writing ratios in an equivalent form, e.g. $6: 8$ can be written as $3: 4$, leading on to the form $1: n$. <br> An interesting alternative to exercise questions is the ratio jigsaw called 'Tarsia - ratio (general)' on the TES website (www.tes.co.uk) which learners can work in groups to complete. A fun homework task would be for learners to produce their own 'ratio' jigsaw. They can produce their own jigsaw using blank equilateral triangles and paper. However, if they prefer to do this task electronically then Tarsia software can be downloaded from the internet. (I) <br> The next step is to look at ratio problems where you are not given the total. For example, 'Two lengths are in the ratio $4: 7$. if the shorter length is 48 cm , how long is the longer length?' <br> Extend this to examples where you are given the difference. For example, 'The mass of two objects are in the ratio $2: 5$. One object is 36 g heavier than the other, what is the mass of each object? |


| Syllabus ref. | Learning objectives | Suggested teaching activities |
| :---: | :---: | :---: |
|  | Increase and decrease a quantity by a given ratio. | The Nrich website (https://nrich.maths.org) has a series of problems of different levels of difficulty. Search for 'ratio and proportion' to obtain a list. They also have a set of problems on 'Ratio, proportion and rates of change' as well as some short problems; search for 'Ratio proportion and rates of change - short problems'. (I) <br> The Maths Is Fun website (www.mathsisfun.com) has a good summary of the difference between direct and inverse proportion; search for 'Directly proportional and inversely proportional'. <br> Two variables are proportional if there is always a constant ratio between them. The constant is called the constant of proportionality. To express the statement " $y$ is directly proportional to $x$ " mathematically, write $y=c x$, where $c$ is the proportionality constant. This can also be written as $y \propto x$. To express the statement " $y$ is inversely proportional to $x$ " mathematically, write an equation $y=\frac{c}{x}$ or " $y$ is directly proportional to $\frac{1}{x}$. This can be demonstrated visually by drawing a graph of $y$ against $\frac{1}{x}$, as per the example here (https://www.desmos.com/calculator/audngjzwg8). <br> There is a lack of an explicit consensus of the role of a negative constant of proportionality. <br> Look at drawing graphs to determine whether two quantities are in direct proportion. Ask learners to solve a variety of problems involving direct proportion by either the ratio method or the unitary method. Look at quantities in inverse proportion, for example, the number of days to perform a job and the number of people working on the job. You will be able to link proportion to measures of rate and scales, for example exchange rates, average speed, density, map scales and other practical examples. For some ideas, read the online blog 'It started with a map, November 2012' from colinbillett (https://colinbillett.wordpress.com). <br> For extended learners provide some good examples and questions on increasing and decreasing a quantity by a given ratio. <br> Use specimen paper for question one which includes percentage change and compound interest questions to check students understanding ( F ). <br> Use specimen paper 4, Q1(a) to check students' ability to apply ratio, percentages and fraction in context and specimen Paper 3, Q4(a) (b) to further check their understanding of ratio in context and also compound interest (F). |
| $\begin{aligned} & \text { C1.12 and } \\ & \text { E1.12 } \end{aligned}$ | Calculate a given percentage of a quantity. | The best starting point is to revise converting between percentages and decimals. You can use examples that require the learners to find percentages of quantities, such as: to find $15 \%$ of $\$ 24$ they would calculate $0.15 \times 24=3.6$ so the answer is $\$ 3.60$. (Remind learners that in money calculations it is conventional to use 2 d.p. for dollar answers). You should |


| Syllabus ref. | Learning objectives | Suggested teaching activities |
| :---: | :---: | :---: |
|  | Express one quantity as a percentage of another. <br> Calculate percentage increase or decrease. <br> Carry out calculations involving reverse percentages, e.g. finding the cost price given the selling price and the percentage profit. | encourage learners to practice mental arithmetic methods too, for example: divide by 10 to find $10 \%$, halve this to find $5 \%$ and add these results to find $15 \%$. <br> Then, use examples to show how to express one quantity as a percentage of another including where there is a mixture of units. <br> Extend the work on finding percentages of quantities to looking at how to calculate percentage increases and decreases. For example, to increase something by $15 \%$ you need to multiply by 1.15 ; to decrease something by $15 \%$ you need to multiply by 0.85 . Provide practice examples. (I) <br> Eliminate the misconception that increasing a quantity by $50 \%$ and then decreasing the resulting quantity by $50 \%$ leads back to the original value. <br> For extended learners you will need to move on to calculations involving reverse percentages. There are two good videos explaining two different approaches for reverse percentages questions: <br> - 'Reverse percentages' on YouTube by 'MrArnoldsMaths' (https://www.youtube.com/watch?v=OQ9T1-0Up6I) <br> - 'Reverse percentages' on the B grade maths website (http://bgrademaths.blogspot.co.uk) Ask learners to compare these methods and to decide which method they think is easier. <br> The STEM learning website (https://www.stem.org.uk/resources) has an excellent lesson on reverse percentages; search for 'Using Percentages to Increase Quantities N7.' The resource makes links between percentages, decimals and fractions and represents percentage increase and decrease as a multiplication and recognises the inverse relationship between increases and decrease. This material tackles the misconception that an increase of $50 \%$ followed by a decrease in $50 \%$ will take you back to the original value. <br> 'Singapore Maths' bar modelling can be used effectively to visualise reverse percentage problems. The Great Maths Teaching Ideas website (www.greatmathsteachingideas.com) includes an introduction on how to use a bar model with basic proportional reasoning problems; search for 'Bar modelling - a powerful visual approach for introducing number topics'. This can be easily extended to include reverse percentage problems. |
| $\begin{aligned} & \text { C1.13 and } \\ & \text { E1.13 } \end{aligned}$ | Use a calculator efficiently. <br> Apply appropriate checks of accuracy. | Start this topic by using examples to show how to estimate the answer to a calculation by rounding each figure in the calculation to 1 significant figure. Learners then check their estimates by doing the original calculation using a calculator. Find good examples and ask learners to practice. (I) |


| Syllabus ref. | Learning objectives | Suggested teaching activities |
| :---: | :---: | :---: |
|  |  | An interesting extension activity, linking this work to percentages, would be to investigate the percentage error produced by rounding calculations using addition/subtraction and multiplication/division. (You would need to explain percentage error beforehand.) |
| C1.14 and E1.14 | Calculate times in terms of the 24hour and 12 -hour clock. <br> Read clocks, dials and timetables. | A basic starting point would be to revise the units used for measuring time, with examples showing how to convert between hours, minutes and seconds. It is useful to use television schedules and bus/train timetables to help with calculations of time intervals, and conversions between 12 -hour and 24 -hour clock formats. <br> Ask learners to work in pairs or small groups to create a timetable for buses or trains running between two local towns. To extend a topic that is relatively easy for more able learners, there is an interesting case study online called 'Scheduling an aircraft' on the Centre for Innovation in Mathematics Teaching website (www.cimt.org.uk). <br> It is useful for learners to look at world time differences and the different time zones. You could ask them to research and annotate a world map with times in various cities assuming it is $12: 00 \mathrm{pm}$ where you live. Times can be found online at timeanddate.com (www.timeanddate.com) by searching for the worldclock. (I) <br> A common error associated with time calculations occurs when learners use a calculator and are given a decimal answer, e.g. '5.3'. Make sure your learners understand that this means ' 5 hours 18 minutes' not ' 5 hours 3 minutes' or ' 5 hours 30 minutes'. You can illustrate this using the online decimal time converter at springfrog.com (www.springfrog.com), which can be found by searching for the 'Convert between hours minutes $\&$ seconds and decimal time' page. |
| C1.15 and E1.15 | Calculate using money and convert from one currency to another. | Use examples to show how to solve straightforward problems involving exchange rates. It is useful for learners if you link this work to syllabus section 2.9 (using conversion graphs). Up-to-date exchange rates can be found from a daily newspaper or online. <br> To check their understanding, learners can then try the past paper question. (F) <br> Resource Plus <br> Skills Pack: Unit conversions <br> The Skills Pack includes lessons on: <br> - converting between simple units of measure <br> - area and volume <br> - compound measures <br> - interpreting travel and conversion graphs. |




## 2 Algebra and graphs

| Syllabus ref. | Learning objectives | Suggested teaching activities |
| :---: | :---: | :---: |
| C2.1 and E 2.1 | Use letters to express generalised numbers and express basic arithmetic processes algebraically. <br> Substitute numbers for words and letters in formulae. <br> Rearrange simple formulae. <br> Construct simple expressions and set up simple equations. <br> Substitute numbers for words and letters in complicated formulae. <br> Construct and transform complicated formulae and equations, e.g. transform formulae where the subject appears twice. | An effective start to this topic is revising basic algebraic notation. For example, $a+a=2 a, b \times c=b c$ (emphasising that $c b$ is the same as $b c$ but that the convention is to write letters in alphabetical order). Also look at simple examples with indices $d \times d=\mathrm{d}^{2}$ and $e \times e \times e \times e=e^{4}$. Explain to learners how to substitute numbers into a formula, including formulae that contain brackets. <br> Ask learners to work in groups to look at the difference between simple algebraic expressions which are often confused. For example, 'Find the difference between $2 x, 2+x$ and $x^{2}$ for different values of $x$ '. Ask learners "is there a number that makes them all equal?" <br> Once the basics are secure move on to transforming simple formulae, for example rearranging $y=a x+b$ to make $x$ the subject. Learners need to understand how to construct simple expressions and equations from word problems. <br> Extension activity: The puzzle 'Perimeter expressions' on the Nrich website (https://nrich.maths.org) is an excellent activity. (I) <br> For extended learners you will need to build on all of the work above. Moving on to more complicated formulae when substituting, for example those with many orders of operations to consider. You can link the work on transforming formulae to the work on solving equations, asking learners to think about the balance method used in both. <br> A useful assessment tool is Paper 21June 2013 Q15. (I) Examples of constructing more complicated equations and expressions can be found in Paper 41 June 2013 Q5 (a)(b)(c). (F) <br> The final step is to explain to learners how to transform complex formulae such as $x^{2}+y^{2}=r^{2}, s=u t+1 / 2 a t^{2}$, expressions involving square roots, etc. You can use a series of examples to illustrate how to transform formulae containing algebraic fractions (with possible links to the work in topic 2.3) for example $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$. <br> The most challenging formula to transform, which deserves time spending on it, is where the subject appears twice. Using examples showing the factorising and dividing through methods, you can discuss the benefits of each. You can link this work to topic 2.2. <br> A good past paper question on this topic is Nov 2012 Q16. (F) |

Syllabus ref.

C2.2 and
E2. 2

Manipulate directed numbers.
Use brackets and extract common factors, e.g.
expand $3 x(2 x-4 y)$
factorise $9 x^{2}+15 x y$
Expand products of algebraic expressions (two brackets only), e.g. expand $(x+4)(x-7)$; for extended include products of more than two brackets, e.g. $(x+4)(x-7)(2 x+1)$

Factorise where possible expressions of the form:
$a x+b x+k a y+k b y$
$a^{2} x^{2}-b^{2} y^{2}$
$a^{2}+2 a b+b^{2}$
$a x^{2}+b x+c$

An important starting point is to revise all aspects of directed numbers with all four operations and link this to positive and negative algebraic terms with the four operations. The inability to deal with negative numbers can otherwise cause an unnecessary stumbling block in algebraic work.

You will need to use examples, with both positive and negative numbers, to illustrate expanding brackets. Start simply with a single term being multiplied over a bracket containing two or more terms. Extend this technique to multiplying two simple linear brackets together for example $(x-3)(x+7)$. It may be useful to build on the grid method of multiplication linked to the partitioning of numbers. Learners may then find it useful to see a $2 \times 2$ algebraic multiplication grid to help with their understanding. It is important to stress the doing and undoing link between expanding brackets and factorising

The next step is to use examples, with both positive and negative numbers, to illustrate factorising simple expressions with one bracket. Explain that factorising is the reverse of expanding.

For extended learners, move on to examples where they will need to find the products of algebraic expressions, for example $\left(x^{2}+3 x-4\right)(x-5)$. Building on the earlier factorising work, use examples to show learners how to factorise three-term quadratic equations, initially where the coefficient of $x^{2}$ is 1 .

Include examples of simple difference of two squares, such as $x^{2}-16$, emphasising that these can be solved using the same method as three-term quadratics bearing in mind that the coefficient of the $x$ term is 0 . There are some good questions to get your learners practicing factorising simple quadratics on slide 5 of the PowerPoint presentation 'Factorising quadratic expressions' on the TES website (www.tes.co.uk).

A really challenging topic is that of factorising quadratics where the coefficient of $x^{2}$ is not 1 . It is worth spending a considerable amount of time on this topic, including revisiting it throughout the course to ensure methods are not forgotten. A higher order thinking skill is to ask learners to compare different methods for tackling a question. This is particularly useful for more able learners. Ask them to compare the two different methods for factorising quadratics of the form $a x^{2}+b x+c$, where $a \neq 1$. The first method can be found on slide 16 of the power point presentation listed above, (which uses splitting the $x$ term into two terms and then factorising by grouping). The second method can be found on Mr Barton Maths website (http://mrbartonmaths.com) in his eBook 'The Maths E-Book of Notes and Examples' in the section 'More factorising quadratics'. He uses a trial and error approach.

Finally, give learners example problems requiring them to factorise the difference of two squares, for example $16 x^{2}-25 y^{2}$. It is also worth mentioning two-term quadratic factorising examples such as $18 x^{2}-24 x$. Emphasise that these are often poorly answered. Point out that because they are quadratics learners often try to use two sets of brackets instead of just the one set of brackets required.

## Suggested teaching activities

## E2.3

(note there
is no C 2.3 )

Manipulate algebraic fractions, e.g.
$\frac{x}{3}+\frac{x-4}{2}, \frac{2 x}{3}-\frac{3(x-5)}{2}, \frac{3 a}{4} \times \frac{9 a}{10}$,
$\frac{3 a}{4} \div \frac{9 a}{10}, \frac{1}{x-2}+\frac{2}{x-3}$
Factorise and simplify rational expressions, e.g. $\frac{x^{2}-2 x}{x^{2}-5 x+6}$

Building on the work on factorising in topic 2.2, show learners how to factorise and simplify rational expressions such as $\frac{x^{2}-2 x}{x^{2}-5 x+6}$.

Provide learners with plenty of examples and questions. It is worth linking this work on simplifying rational expressions to the work on using the four rules with algebraic fractions, so that learners always give the most simplified answer. (I)

You will need to spend time revising adding and subtracting simple fractions with learners, for example $\frac{2}{5}+\frac{3}{8}$. Explain the process of finding a common denominator by, in this case, multiplying the two denominators. Ask learners to discuss when the lowest common denominator doesn't need to be the product of the two denominators, e.g. $\frac{3}{10}+\frac{5}{8}$.

The next step is to move on to algebraic fractions starting with numerical denominators, for example $\frac{x}{3}+\frac{x-4}{2}$, $\frac{2 x}{3}-\frac{3(x-5)}{2}$ then extending this to algebraic denominators such as $\frac{1}{x-2}+\frac{2}{x-3}$. You will need to emphasise common errors that occur when subtracting algebraic fractions. For example, in $\frac{3}{x-5}-\frac{4}{x+2}$ explain that it is common to see sign errors on the numerator when $x-5$ is multiplied by -4 .

Move on to examples demonstrating multiplying and dividing with numerical fractions, reminding learners that instead of dividing by a fraction you multiply by its reciprocal.

Extend this by looking at algebraic fractions such as $\frac{3 a}{4} \times \frac{9 a}{10}, \frac{3 a}{4} \div \frac{9 a}{10}$.
Provide example questions for learners to practice. Paper 21 June 2013 Q22 is also worth doing. (I) (F)

Syllabus ref.

## C2.4 ad

E2.4
Use and interpret positive, negative and zero indices.

Use the rules of indices, e.g. simplify $3 x^{4} \times 5 x, 10 x^{3} \div 2 x^{2},\left(x^{6}\right)^{2}$ and for extended learners, e.g. $3 x^{-4} \times \frac{2}{3} x^{\frac{1}{2}}$, $\frac{2}{5} x^{\frac{1}{2}} \div 2 x^{-2},\left(\frac{2 x^{5}}{3}\right)^{3}$

Use and interpret fractional indices, e.g. $32^{x}=2$

## C2.5 and

E2.5

Derive and solve simple linear equations in one unknown.

Derive and solve simultaneous linear equations in two unknowns.

Derive and solve simultaneous equations, involving one linear and one quadratic.

Derive and solve quadratic equations by factorisation, completing the square and by use of the formula.

Derive and solve linear inequalities, including representing and interpreting inequalities on a number line; interpretation of results may be required.

A good starting point is to give learners examples that revise the rules of indices work from Unit 1 topic 1.7. Extend this to using and interpreting positive, negative and zero indices and using the rules of indices with algebraic terms. For example, simplify: $3 x^{4} \times 5 x, 10 x^{3} \div 2 x^{2},\left(x^{6}\right)^{2}$.

For extended learners, move on to looking at fractional indices. For example, simplify: $3 x^{-4} \times \frac{2}{3} x^{\frac{1}{2}}, \frac{2}{5} x^{\frac{1}{2}} \div 2 x^{-2},\left(\frac{2 x^{5}}{3}\right)^{3}$, and solving exponential simple equations such as, $32^{x}=2$.

Begin this work with revising how to solve simple linear equations, including those with negatives, for example $3 x+2=-1$. You should also include examples showing how to solve linear equations with brackets such as $5(x+4)=3(x+10)$.

For a fun active learning resource, ask learners to work in groups to complete the 'Simple equations jigsaw' activity from the TES website (www.tes.com . Many more jigsaws are available at Mr Barton Maths website (http://mrbartonmaths.com), which also contains the link for downloading the Tarsia software to view the jigsaws.

Simultaneous equations can be set up for a range of real life problems. A good introduction to simultaneous equations is to use a non-algebraic approach that builds on learners' informal approaches to these problems. For example, 3 coffees and 2 teas cost $\$ 6.50$, and 5 coffees and 2 teas cost $\$ 9.50$. Showing learners how the simultaneous equation from these statements can be formed and emphasising that the cost of tea and coffee does not change. You can start with concrete examples and visual images to build on learners informal understanding. It is important for learners to understand that to solve problems that involve two unknowns it is necessary for them to have two equations. The aim of solving simultaneous equations is to remove one of the unknowns, then they can approach the problem using what they already know about solving simple linear equations in one unknown.

Extend this by looking at examples to illustrate how to solve simultaneous linear equations with two unknowns by elimination, substitution and finding approximate solutions using graphical methods (linking to topic 2.10). You can use software packages such as Desmos (www.desmos.com) or Geogebra Graphing Calculator (www.geogebra.org) to allow learners to explore the solutions of simultaneous equations graphically. (I)

Extended learners need solve simultaneous equations involving one linear and one quadratic equation. They can also be asked to compare the two methods for solving simultaneous equations (elimination and substitution) and discuss which methods they might use and why for specific examples. In many cases, substitution may be more appropriate when a quadratic equation is involved.

Extended learners will then need to explore all the different methods for solving quadratic equations, namely by factorisation, using the quadratic formula and completing the square (for real solutions only). The Maths is Fun website (www.mathsisfun.com) has a good explanation of completing the square that uses multiple representations to help secure learners conceptual understanding of the process. (If the link breaks, search 'Completing the square'.)

The best starting point is using examples of the form $a x^{2}+b x+c=0$ then extend this by looking at equations requiring rearranging into this form first. Paper 41 June 2017 Q8(b), is a good example. (F)(I)

A more challenging activity involves learners needing to construct their own equations from information given and then solve them to find the unknown quantity or quantities. This could involve the solution of linear equations, simultaneous equations or quadratic equations.

To introduce the topic of solving linear inequalities, it is a good idea starting with just numbers, for example $7>5$, showing that that multiplying or dividing an inequality by a negative number reverses the inequality sign, i.e. $-7<-5$.

Use examples to illustrate how to solve simple linear inequalities including representing the inequality on a number line. Interpretation of results may be required. You can use software packages such as Geogebra (www.geogebra.org) to create interactive activities for learners to explore and demonstrate their understanding of inequalities on a number line. You can also explore the resources on the Geogebra website for some examples by searching for inequalities on a number line.

## (I)(F)

The most challenging inequalities for learners to solve are those where the inequality needs to be split into two parts and each part solved separately. Paper 22 June 2017 Q13 and Paper 23 June 2017 Q16 are good examples of inequalities questions. (F)

## Suggested teaching activities

## E2.6

(note there
is no C2.6)
$\square$

## C2.7 and

E2. 7

Represent inequalities graphically and use this representation to solve simple linear programming problems.

The conventions of using broken lines for strict inequalities and shading unwanted regions will be expected.

## Continue a given number sequence.

Recognise sequences of square, cube and triangular numbers.

Recognise patterns in sequences including the term to term rule and relationships between different sequences; subscript notation might be used.

Find and use the $n$th term of sequences in linear, simple quadratic and cubic sequences; for extended learners this is required for linear, quadratic, cubic and exponential sequences and simple combinations of these.

A good starting point is to begin by asking learners to draw several straight lines on a set of axes, possibly on mini white boards, for example $y=2, x=-5, y=3 x$ and $x+2 y=10$. Ask learners to consider a point on one side of each of these lines, the origin if possible, and use substitution to see if the inequalities $y<2, x\rangle-5, y<3 x$ and $x+2 y>10$ are true for their chosen point. Ask learners to work in groups to do their own examples.

Extend this work by asking learners to look at examples illustrating how to solve linear programming problems by graphical means, highlighting the convention of using broken lines for strict inequalities < and > and solid lines for the inequalities $\leq$ and $\geq$.

Finally, learners will need to understand how to construct inequalities from constraints given, showing that several possible solutions to a problem exist, indicated by the unshaded region on a graph. Provide learners with examples and questions.

Give learners the definition of a sequence of numbers. Begin by asking them to work in groups to investigate some simple sequences, such as finding the next two numbers in a sequence of even, odd, square, triangle or Fibonacci numbers.

Extend this to looking at finding the term-to-term rule for a sequence. For example, the sequence $3,9,15,21,27, \ldots$, has a term-to-term rule of +6 ; the sequence $40,20,10,5,2.5, \ldots$, has a term to term rule of $\div 2$. Learners will need to have some appreciation of the limitations of a term-to-term rule, i.e. that they are not very useful for finding terms that are a long way down the sequence. This leads on nicely to finding the position-to-term rule for a sequence by examining the common difference, for example the $n$th term in the sequence $3,9,15,21,27, \ldots$, is $6 n-3$.

The Nrich website (https://nrich.maths.org) has a nice activity called 'Seven squares - group worthy task' that looks at challenges learners to describe generic patterns verbally, numerically and algebraically. It does not assume prior knowledge of algebra and could be a good way to introduce, practise or assess algebraic fluency. You could use mini whiteboards or coloured matchsticks to support this activity. (F)

An interesting investigation is to look at square tables placed in a row so that 4 people can sit around one table, 6 people can sit around 2 tables joined, 8 people can sit around 3 tables joined, and so on. Ask learners to work out how many people can sit around $n$ tables. To add an extra challenge, ask learners to investigate tables of different shapes and sizes, and to try to relate the $n$th term formula to the practical situation explaining how the numbers in the formula relate to the arrangements of the tables. (I)

Extension activity: With more able learners you could look at deriving the formula for a linear sequence with $n$th term $=$ $a+(n-1) d$ where $a$ is the first term and $d$ is the common difference, this formula is not essential knowledge. (I)

| Syllabus ref. | Learning objectives | Suggested teaching activities |
| :---: | :---: | :---: |
|  |  | Another approach is looking at patterns and relationships between different sequences. For example, the sequence $2,5,10$, $17,26, \ldots$, is the square numbers +1 . You can give learners several examples of these asking them to find the $n$th term, using just simple quadratic and cubic sequences, i.e. of the form $a n^{2} \pm c$ or $a n^{3} \pm c$. (I) <br> Extended learners, can explore examples of finding the $n$th term of harder quadratic sequences and can go on to investigate cubic sequences. <br> Other methods for finding $n$th terms are possible. Ask learners to search online for alternative methods for finding $n$th terms. (I) <br> Subscript notation may be used. The introduction to sequences on the Maths is Fun website (http://www.mathsisfun.com) includes an explanation of subscript notation. Search for 'Sequences'. <br> Finally, learners will need to look at exponential sequences with a common multiplier (or ratio) instead of a common difference. <br> Extension activity: With more able learners, derive the formula for the $n$th term $=a r^{(n-1)}$ where $a$ is the first term and $r$ is the common ratio, this formula is not essential knowledge. |
| E2.8 <br> (note there is no C 2.8 ) | Express direct and inverse proportion in algebraic terms and use this form of expression to find unknown quantities. | Learners will need to be able to solve a variety of problems involving direct or inverse variation. <br> Encourage efficient notation that moves from the question to each step in turn. For example, <br> - $y$ varies directly with $x$ (or $y$ is directly proportional to $x$ ) to $y \propto x \Rightarrow y=k x$ <br> - $t$ varies inversely as the square root of $v$ to $v \Rightarrow t \propto \frac{1}{v^{2}} \Rightarrow t=\frac{k}{v^{2}}$ where $k$ is a constant. |

Syllabus ref.

|  |  |
| :--- | :--- |
| E2.9 <br> (note there <br> is no C2.9) | Use function notation, e.g. <br> $\mathrm{f}(x)=3 x-5, \mathrm{f}: x \mapsto 3 x-5$, to describe <br> simple functions. <br> Find inverse functions $\mathrm{f}-1(x)$. |
|  | Form composite functions as defined <br> by $\operatorname{gf}(x)=\mathrm{g}(\mathrm{f}(x))$. |

Emphasise the common error of reversing direct and inverse variation. Once the formula has been established ask learners to use given values to work out the value of the constant, $k$, and then use the formulae with the evaluated $k$.

Give learners a definition of a function, $\mathrm{f}(x)$ : that it is a rule applied to values of $x$. Look at evaluating simple functions for specific values, for example linear functions, describing the functions using $\mathrm{f}(x)$ notation and mapping notation.

The next step is to introduce the inverse function as an operation which 'undoes' the effect of a function. Demonstrate how learners can evaluate simple inverse functions for specific values, describing the functions using the $\mathrm{f}^{-1}(x)$ notation and mapping notation. Link this to the work on transforming formulae from topic 2.1. Explain to learners that to find the inverse of the function $\mathrm{f}(x)=2 x-5$, a useful method is to rewrite this as $y=2 x-5$, then to interchange the $x$ and $y$ to get $x=2 y-5$, then to make $y$ the subject $y=(x+5) / 2$ and finally to re-write using the inverse function notation as $\mathrm{f}^{-1}(x)=(x+5) / 2$.

Using linear and/or quadratic functions, $\mathrm{f}(x)$ and $\mathrm{g}(x)$, show learners how to form composite functions such as $\mathrm{gf}(x)$, and how to evaluate them for specific values of $x$. Ask learners to investigate for a variety of different functions $\operatorname{gf}(x)$ and $\mathrm{fg}(x)$ to see that these are not often the same. Emphasise that it is important that learners know the correct order to apply the functions.

Provide learners with examples and questions, either prepared yourself or from textbooks.
Extension activity: The video 'Finding inverse functions: linear' on the Kahn academy website (www.khanacademy.org) also talks about what the graph of an inverse function looks like. Knowing that the graph of an inverse function is a reflection in the line $y=x$ is is a useful extension for the more able learners.

A good starting point is to draw and use straight line graphs to convert between different units, for example between metric and imperial units, or between different currencies. Exchange rates can be found online and can be useful for setting questions. Learners need to be confident in solving problems using compound measures. It will be useful to link this work to the work from topic 1.11 and 1.15.

It is important for learners to be able to draw a variety of graphs from given data, for example to determine whether two quantities are in proportion, e.g. $y$ and $x$ (or for more able learners $y$ and $x^{2}$ ). You will be able to link this to the work in topic 2.8 on direct and inverse variation (for extended learners).

## Suggested teaching activities

Apply the idea of rate of change to simple kinematics involving distancetime and speed-time graphs, acceleration and deceleration; may include estimation and interpretation of the gradient of a tangent at a point.

Calculate distance travelled as area under a speed-time graph.

## C2.11

Construct tables of values for functions of the form $a x+b, \pm x^{2}+a x+b$,
$\frac{a}{x}(x \neq 0)$, where $a$ and $b$ are integer constants.

Draw and interpret such graphs.
Solve linear and quadratic equations approximately, including finding and

For extended learners, you will want to provide examples of how to draw and use distance-time graphs to calculate average speed (linking this to the calculating gradients work in topic 5.2). Learners should be able to interpret the information shown in travel graphs and to be able to draw travel graphs from given data. Ask learners to draw a travel graph for an imaginary journey and to write a set of questions about this journey. For example, "What was the average speed?" (I) When learners have drawn their graphs and written their question they can then give these to other members of a group to answer.

The STEM learning e-library (https://www.stem.org.uk/resources) has an example of a lesson that can be used to deepen or assess learners understanding in this section. Search for 'Interpreting distance-time graphs A6'. (I)(F)
You will need to ensure that learners have studied topic 4.2 and that they can confidently calculate areas of rectangles, triangles, trapeziums and compound shapes derived from these.

Extend this work by looking at examples of speed-time graphs being used to find acceleration and deceleration and to calculate distance travelled as area under a linear speed-time graph.

Challenge can be provided by looking at examples where learners are required to convert between different units. For example, where different units are being used in the question and in the graph.

## Resource Plus

## Skills Pack: Unit conversions

The Skills Pack includes lessons on:

- converting between simple units of measure
- area and volume
- compound measures
- interpreting travel and conversion graphs.

Begin this topic by drawing a series of lines with $x=$ constant and $y=$ constant. Ask learners to identify the equations of the lines that you have drawn. Emphasise the importance of using a ruler and a sharp pencil in mathematical diagrams throughout this topic.

Move on to examples of drawing diagonal straight-line graphs from a table of values where the gradient and intercept are integers. You can link this to the work on gradient in topic 5.2.

Extension activity: 'Graphing linear equations' is an online lesson from math.com (http://www.math.com) where learners can work as a group to explore and compare the methods for drawing lines from equations.
interpreting roots by graphical methods.

Recognise, sketch and interpret graphs of functions; linear and quadratic only. Knowledge of turning points is not required.

## E2.11

Construct tables of values and draw graphs for functions of the form $a x^{n}$ (and simple sums of these) and functions of the form $a b^{x}+c$, where $a$ and $c$ are rational constants, $b$ is a positive integer, and $n=-2,-1,0,1,2$, 3 ; sums would not include more than three functions.

Solve associated equations approximately, including finding and

Extend this to looking at drawing quadratic functions of the form $\pm x^{2}+a x+b$, and simple reciprocal functions such as $\frac{a}{x}(x \neq 0)$. Learners should be able to draw a variety of these graphs confidently and accurately from a table of values.

Introduce the terms parabola and hyperbola (although these are not required).
You can then discuss with learners the symmetry properties of a quadratic graph and how this is useful; knowledge of turning points is not required.

The STEM learning e-library (https://www.stem.org.uk/resources) has an example of a good lesson that could be used to consolidate or assess learners' ability to identify and interpret different graphs. Search for 'Interpreting functions graphs and tables'. Note the software mentioned in the lesson is not necessary for the activity but can enrich it if available. (I)(F)

The next step is to show how the solutions to a quadratic equation may be approximated using a graph. Extend this work to show how the solution(s) to pairs of equations (for example $y=x^{2}-2 x-3$ and $y=x$ ) can be estimated using a graph. This work can be linked to the work on simultaneous equations from topic 2.5 .

Software drawing packages such as Geogebra (www.geogebra.org) are useful for learners to use to investigate different features of graphs. Geogebra is free to download.

Start by asking learners to draw functions of the form $a / x^{2} ; a / x ; a x^{3} ; a^{x}$; where $a$ is a constant, using a graph drawing package like Geogebra. Ask learners to work in groups to use the software to gain an awareness of what each of the different types of graphs look like. Learners should recognise common types of functions from their graphs, for example from the parabola, hyperbola, quadratic, cubic and exponential graphs.

The Maths is Fun website (https://www.mathsisfun.com) can be used to explore the effect of transforming graphs. Search for 'Function transformations'. This page explains how all transformations can be done in one go using the arrangement:
interpreting roots by graphical methods; Find turning points of quadratics by completing the square.

Draw and interpret graphs representing exponential growth and decay problems.

Recognise, sketch and interpret graphs of functions; linear, quadratic, cubic, reciprocal and exponential.

Knowledge of turning points and asymptotes is required.
$a \mathrm{f}(b(x+c))+d$. Remind students that for quadratic equations completing the square arranges the quadratic function in this form.

GeoGebra can be used to allow learners to explore transformations of functions independently, and you can also find existing examples in the resource section of the Geogebra website. (I)

Once learners understand the effect of transforming a quadratic equation written in the form $a \mathrm{f}(b(x+c))+d$, they will be able to derive how they find the turning point for different quadratic equations by completing the square and relating this to a transformation of the graph of $x^{2}$.

The STEM learning e-library (www.stem.org.uk) has a good resource to consolidate or assess students understanding of how to use transformations of the graph of $x^{2}$ to identify key properties such as turning points; it is called 'Linking the properties and forms of quadratic functions $\mathrm{C} 1^{\prime}$. (I) (F)

The Maths is Fun website (www.mathsisfun.com) also has a useful page describing different asymptotes. This includes questions for learners to explore on their own, including detailed explanations to support the solutions. Learners could explore these questions alongside the use of GeoGebra graphing calculator package. (I)

Move on to asking learners to draw the graphs from tables of values. A useful video 'Exponential function graph' can found on the Khan academy website (www.khanacademy.org). Extend the work to include simple sums of not more than three functions in the form $a x^{n}$, where $a$ is a rational constant, and $n=-2,-1,0,1,2,3$. Ask learners to solve associated equations approximately using these graphs.

Learners should be encouraged to sketch a range of graphs by recognising key points on these graphs. They should realise that sketching a graph is different from drawing a graph and that both are useful; they may be asked to do both. Learners should understand that a sketch of a graph does not need to be $100 \%$ accurate and to scale; it is important however, that the most important features are there and clearly labelled. Questions that they could be encouraged to ask themselves should include:

- What happens when $x=0$ ? When $y=0$ ?
- What happens when $x$ tends towards infinity?
- Are there any asymptotes? Horizontal? Vertical? Oblique?
- They could also link this to trigonometrical functions by considering whether the graph is going to be periodic. (Links to topic E6.3).

The final step is to look at examples of how to draw and interpret graphs representing exponential growth and decay problems. It will be useful to learners to link this to the work from topic 1.17.

| $\underset{\text { re }}{\text { Sylla }}$ | Learning objectives | Suggested teaching activities |
| :---: | :---: | :---: |
| E2.12 | Estimate gradients of curves by drawing tangents. | Ensure learners have studied topic 5.2 (finding the gradient of a straight line) before beginning this topic. <br> Learners should already be able to confidently find the gradient of a straight line. Give learners a definition of the term tangent. Move on to looking at examples that show how to find the gradient at a point on a curve by drawing a tangent at that point. |
| E2.13 | Understand the idea of a derived function. <br> Use the derivatives of functions of the form $a x^{n}$, and simple sums of not more than three of these, where $a$ is a rational constant and $n$ is a positive integer or 0 . <br> Apply differentiation to gradients and turning points (stationary points), e.g. $2 x^{3}+x-7$ <br> Discriminate between maxima and minima by any method. | Link this work to work learners have already done on the gradient of a straight line graph and distance-time graphs. Remind them about the equation for finding the gradient of a straight line. <br> The Maths is Fun website (www.mathsisfun.com) provides good introductions to calculus. Search 'Introduction to Calculus' and 'Introduction to derivatives'. <br> Introduce learners to this general formula and use this to explore what happens to functions of the form $a x^{n}$ as the change in $x$ tends towards zero: $\frac{\square y}{\square x}=\frac{f(x+\square x)-f(x)}{\square x}$ <br> The STEM learning e-library ( www.stem.org.uk) has a good resource that could be used to consolidate or assess students understanding of derivatives of functions of the form $a x^{n}$. Search 'Matching functions and derivatives' (I)(F) |
| Past and specimen papers |  |  |
| Past/s <br> E 2.1 <br> C2.2: <br> E2.2: <br> E2.3: <br> E2.4: <br> C2.5: | n papers and mark schemes are available <br> 3 June 2017 Q23; Paper 42 Nov 2017Q8 <br> 21 June 2017Q5 <br> 22 June 2017 Q22(b) <br> 23 Nov 2017 Q2( c); Paper 21 Nov 2017 <br> 2 June 2017 Q22(a) <br> 43 June 2017 7(a); Paper 22, Nov 2016 Q | the $\mathbf{0 5 8 0}$ syllabus to download at www.cambridgeinternational.org/support ( $\mathbf{F}$ ) |

Syllabus
ref.

E2.5: Specimen Paper 2 Q28; Paper 41 June 2017 Q8b (Quadratics); Paper 22 June 2017 Q13 (Inequality); Paper 23 June 2017 Q16(Inequality)
E2.6: Paper 23, June 2017 Q11
C2.7: Paper 41, June 2017, Q9
E2.7: Paper 43, Nov 2016, Q10; Specimen Paper 2 Q15
E2.8: Paper 22 June 2017 Q21
E2.9: Specimen Paper 4 Q7; Paper 41 Nov 2017 Q4; Paper 23 June 2017 Q12
E2.11: Specimen Paper 4 Q3(iv)
E2.13: Specimen Paper 4 Q11

## 3 Coordinate geometry

| Syllabus <br> ref. | Learning objectives |  |
| :--- | :--- | :--- |
| C3.1 and <br> E3.1 | Demonstrate familiarity with Cartesian <br> coordinates in two dimensions. | Revise coordinates in two dimensions. Draw a picture by joining dots on a square grid. Draw $x$ and $y$ axes on the grid and <br> write down the coordinates of each dot. (I) <br> Ask other learners to draw these pictures from a list of coordinates only. |
| C3.2 and <br> E3.2 | Find the gradient of a straight line. <br> Calculate the gradient of a straight line <br> from the coordinates of two points on <br> it. | Use a diagram to help you define a line with a positive gradient as one sloping upwards, and a line with a negative <br> gradient as one sloping downwards. <br> Use simple examples to show how to calculate the gradient (positive, negative or zero) of a straight line from a graph <br> using vertical distance divided by horizontal distance in a right-angled triangle: |
| gradient = $\frac{\text { change in } y \text { coordinates }}{\text { change in } x \text { coordinates }}$ |  |  |


| Syllabus ref. | Learning objectives | Suggested teaching activities |
| :---: | :---: | :---: |
| E3.3 <br> (note there is no C3.3) | Calculate the length and the coordinates of the midpoint of a straight line from the coordinates of its end points. | Revise Pythagoras' theorem from Unit 4. Use examples to show how to calculate the length of a straight line segment from the coordinates of its end points using a sketch. <br> Extension activity: To challenge the learners, do this using the formula $\sqrt{\left(x_{1}-x_{2}\right)^{2}+\left(y_{1}-y_{2}\right)^{2}}$. <br> Use examples to show how to find the coordinates of the midpoint of a straight line from the coordinates of its end points. Include examples working backwards, e.g. when an end point and a midpoint are known, find the other end point. (I) |
| C3.4 and E3.4 | Interpret and obtain the equation of a straight line graph in the form $y=m x+c$. <br> Problems will involve finding the equation where the graph is given. | Revise drawing a graph of $y=m x+c$ from a table of values. Interpret the meaning of $m$ and $c$ from the equation using the terms gradient and intercept. Starting with a straight line graph, show how its equation $(y=\mathrm{m} x+\mathrm{c})$ can be obtained. (I) <br> To interpret the meaning of an equation, explain how an equation simply gives the relationship between the $x$ and $y$ coordinates on the line, e.g. for the equation $y=2 x$ this means the $y$ ordinate is always double the $x$ ordinate. Use this to identify if a point lies on the line, e.g. which of these points: $(2,8),(-4,8),(7,14),(20,10),(0,0)$ lies on the line $y=2 x$ ? <br> Ask learners to come up similar questions. (I) Then give these questions to others in a group to identify which points do not lie on a given line. |
| C3.5 and E3.5 | Determine the equation of a straight line parallel to a given line. <br> e.g. find the equation of a line parallel to $y=4 x-1$ that passes through $(0,-3)$. | Use examples to show how to find the equation of a straight line parallel to a given line, e.g. find the equation of a line parallel to $y=4 x-1$ that passes through $(0,-3)$. <br> The Skills Pack includes a lesson on determining the equation of a straight line parallel to a given line. |

Syllabus ref.

## Learning objectives

Find the gradient of parallel and perpendicular lines, e.g.
find the gradient of a line perpendicular
to $y=3 x+1$
find the equation of a line perpendicular to one passing through the coordinates $(1,3)$ and $(-2,-9)$.

Use examples to show that parallel lines have the same gradient. Include examples where the equation is given implicitly, e.g. which of these lines are parallel? $y=2 x, y+2 x=10, y-2 x+3,2 y=2 x+7$, etc.

Use an odd-one-out activity with three or more examples, where one of the lines is not parallel to the others and ask learners to identify which one is the odd-one-out and why. Ask learners to come up with their own set of odd one out examples.

Find the gradient of perpendicular lines by using the fact that if two lines are perpendicular the product of their gradients is -1 , e.g. find the gradient of a line perpendicular to $y=3 x+1$.

Use a variety of examples linking earlier topics from this unit, e.g. find the equation of a line perpendicular to one passing through the coordinates $(1,3)$ and $(-2,-9)$.

You could use the following resources to assess learners' understanding of this objective along with objective E.3.2 and C3.4) (I) (F):

- Parallel lines: http://www.mathsisfun.com/algebra/line-parallel-perpendicular.html
- Lots of lines!: https://undergroundmathematics.org/geometry-of-equations/lots-of-lines


## Resource Plus

Skills Pack: Straight line graphs
The Skills Pack includes a lesson on finding the gradient of parallel and perpendicular lines.

## Past and specimen papers

Past/specimen papers and mark schemes are available for the $\mathbf{0 5 8 0}$ syllabus to download at www.cambridgeinternational.org/support ( $\mathbf{F}$ )
E3.2: Paper 41 June 2017 Q7(a)
C3.4: Specimen Paper 3 Q7(a); Specimen Paper 2 Q5
E3.6: Paper 41 June 2017 Q7 (c) and (d)

## 4 Geometry

| Syllabus ref. | Learning objectives Suggested teaching activities |  |
| :---: | :---: | :---: |
| C4.1 and E4.1 | Use and interpret the geometrical terms: point, line, parallel, bearing, right angle, acute, obtuse and reflex angles, perpendicular, similarity and congruence. <br> Use and interpret vocabulary of triangles, quadrilaterals, circles, polygons and simple solid figures including nets. | Use flashcards at Quizlet (https://quizlet.com) to look at the geometrical terminology. <br> Introduce the terminology for bearings, similarity and congruence briefly as similar shapes and three-figure bearings will be studied in more detail in topics 3.4 and 6.1. <br> Illustrate common solids, e.g. cube, cuboid, tetrahedron, cylinder, cone, sphere, prism, pyramid. Define the terms vertex, edge and face. <br> Explore some geometric solids and their properties on the Illuminations website (https://illuminations.nctm.org )(I); search for 'Geometric solids'. |
| C4.2 and E4. 2 | Measure and draw lines and angles. <br> Construct a triangle given the three sides using a ruler and a pair of compasses only. | Reinforce accurate measurement of lines and angles through various exercises. For example, each learner draws two lines that intersect. Measure the length of each line to the nearest millimetre and one of the angles to the nearest degree. Each learner should then measure another learner's drawing and compare answers. <br> Ask learners to draw any triangle and then measure the three angles and check that they add up to $180^{\circ}$. <br> Show how to construct a triangle using a ruler and a pair of compasses only, given the lengths of all three sides. The Maths is fun website (www.mathsisfun.com) has a useful animation, 'Constructing a triangle with 3 known sides', to demonstrate. |


| Syllabus <br> ref. | Learning objectives |  |
| :--- | :--- | :--- |
| C4.3 and <br> E4.3 | Read and make scale drawings. | Use an example to revise the topic of scale drawing. <br> Show how to calculate the scale of a drawing given a length on the drawing and the corresponding real length. Point out <br> that measurements do not need to be included on a scale drawing and that many scale drawings usually have a scale <br> written in the form $1: n$. |
| Draw various situations to scale and interpret results. For example, ask learners to draw a plan of a room in their house to |  |  |
| scale and use it to determine the area of carpet needed to cover the floor, plan an orienteering course, etc. (I) |  |  |
| Explore the set of resources on the Khan Academy website (https://www.khanacademy.org/) by searching for 'scale |  |  |
| drawings'. |  |  |


| Syllabus ref. | Learning objectives | Suggested teaching activities |
| :---: | :---: | :---: |
| C4.5 and E4.5 | Recognise congruent shapes. <br> Use the basic congruence criteria for triangles (SSS, ASA, SAS, RHS). | Discuss the conditions for congruent triangles. Point out that when naming triangles that are congruent, it is usual to state letters in corresponding order. For example, stating that $\triangle A B C$ is congruent to $\triangle E F G$ implies that the angle at $A$ is the same as the angle at $E$. <br> Extend the work on congruent shapes to introduce similar triangles/shapes. Use the fact that corresponding sides are in the same ratio to calculate the length of an unknown side. Link this work to work on transformations since rotation, reflection and translation leave shapes congruent and enlargements form similar shapes. <br> The Khan Academy website (https://www.khanacademy.org) has a number of resources under 'Triangle congruence' that include the basic congruent criteria. |
| C4.6 | Recognise rotational and line symmetry (including order of rotational symmetry) in two dimensions. Includes properties of triangles, quadrilaterals and circles directly related to their symmetries. <br> Recognise symmetry properties of the prism (including cylinder) and the pyramid (including cone). <br> Use the following symmetry properties of circles: <br> - equal chords are equidistant from the centre <br> - the perpendicular bisector of a chord passes through the centre <br> - tangents from an external point are equal in length. | Define the terms line of symmetry and order of rotational symmetry for two dimensional shapes. Revise the symmetries of triangles (equilateral, isosceles) and quadrilaterals (square, rectangle, rhombus, parallelogram, trapezium, kite) including considering diagonal properties. Discuss the infinite symmetry properties of a circle. <br> You can use classifying and 'odd-one-out' activities to engage learners with comparing and contrasting the properties of these shapes related to their symmetries. An example of this type of activity can be found on the STEM e-library (www.stem.org.uk); search for 'Classifying shapes SS1'. <br> For extended learners, define the terms plane of symmetry and order of rotational symmetry for three dimensional shapes. Use diagrams to illustrate the symmetries of cuboids (including a cube), prisms (including a cylinder), pyramids (including a cone). Look at diagrams for the symmetry properties of a circles paying attention to chords and tangents. |

Syllabus ref.

Learning objectives

Calculate unknown angles using the
following geometrical properties:

- angles at a point
- angles at a point on a straight line and intersecting straight lines
- angles formed within parallel lines
- angle properties of triangles and quadrilaterals
- angle properties of regular polygons
- angle in a semicircle
- angle between tangent and radius of a circle
- angle properties of irregular polygons
- angle at the centre of a circle is twice the angle at the circumference
- angles in the same segment are equal
- angles in opposite segments are supplementary; cyclic quadrilaterals
- alternate segment theorem.

Candidates will be expected to use the correct geometrical terminology when giving reasons for answers.

Revise basic angle properties by drawing simple diagrams that illustrate angles at a point; angles on a straight line and intersecting lines; angles formed within parallel lines and angle properties of triangles and quadrilaterals.

Define the terms irregular polygon, regular polygon, concave and convex. Use examples that include: triangles, quadrilaterals, pentagons, hexagons and octagons. Show that each exterior angle of a regular polygon is $360^{\circ} / n$, where $n$ is the number of sides, and that the interior angle is $180^{\circ}$ minus the exterior angle. Solve a variety of problems that use these formulae. Draw a table of information for regular polygons. Use as headings: number of sides, name, exterior angle, sum of interior angles, interior angle. (I)

Use diagrams to show the angle in a semicircle and the angle between tangent and radius of a circle are $90^{\circ}$. Use the dynamic pages on timdevereux.co.uk to see the circle theorems come to life.

Provide the solution to an examination style question on the topic of angles that contains a mistake in the working. Ask learners to identify the mistake.

For extended learners, move on to look at angle properties of irregular polygons. By dividing an $n$-sided polygon into several triangles, show that the sum of the interior angles is $180(n-2)$ degrees and that the interior and exterior angles sum to $180^{\circ}$.

Explain the theory that angles in opposite segments are supplementary. Investigate cyclic quadrilaterals. For example, explain why all rectangles are cyclic quadrilaterals. What other quadrilateral is always cyclic? Is it possible to draw a parallelogram that is cyclic?, etc. Use examples to show that the angle at the centre of a circle is twice the angle at the circumference and that angles in the same segment are equal.

Introduce learners to the process of proof by demonstrating one of the circle theorems and then asking them to reproduce the proof independently, or by creating a proof and then cutting it up and asking learners to reconstruct it. This second approach can be made more challenging by leaving steps out of the proof for learners to identify and complete. You could also ask learners to provide feedback on exemplars. (I)

Solve a variety of problems using all the circle theorems making sure that learners know the correct language for describing the reasoning for their answers.

## Syllabus

ref.

Past and specimen papers
Past/specimen papers and mark schemes are available for the $\mathbf{0 5 8 0}$ syllabus to download at www.cambridgeinternational.org/support (F)
C4.2: Specimen Paper Q10
C4.4: Paper 12 June 2017 Q18
E4.4: Paper 21 June 2017 Q11; Paper 21 Nov 2017 Q20
C4.5: Specimen Paper 3 Q9(c)
C4.7: Paper 21 June 2017 Q14
E4.7: Paper 22 June 2017 Q 26; Specimen Paper 2 Q26

## 5 Mensuration

| Syllabus ref. | Learning objectives | Suggested teaching activities |
| :---: | :---: | :---: |
| C5.1 and E5.1 | Use current units of mass, length, area, volume and capacity in practical situations and express quantities in terms of larger or smaller units. <br> Convert between units including units of area and volume. | A good starting point is to use practical examples to illustrate how to convert between: millimetres, centimetres, metres and kilometres; grams, kilograms and tonnes; millilitres, centilitres and litres. For example, by looking at various measuring scales. <br> Extend this work to look at converting between units of area $\mathrm{mm}^{2}, \mathrm{~cm}^{2}$ and $\mathrm{m}^{2}$ and volume $\mathrm{mm}^{3}, \mathrm{~cm}^{3}$ and $\mathrm{m}^{3}$. <br> More able learners will probably find it interesting to explore the link between the work on converting between area units to the work on ratio and similar shapes, and can look at using scales on maps to work with areas. <br> Resource Plus <br> Skills Pack: Unit conversions <br> The Skills Pack includes lessons on: <br> - converting between simple units of measure <br> - area and volume <br> - compound measures <br> - interpreting travel and conversion graphs. |
| C5.2 | Carry out calculations involving the perimeter and area of a rectangle, triangle, parallelogram and trapezium and compound shapes derived from these. | Begin this topic by reminding learners how to calculate the perimeter and area of a rectangle, square and a triangle. This can be extended to look at how to calculate the area of a parallelogram and a trapezium, and a variety of compound shapes. <br> An interesting investigation is to look at using isometric dot paper to find the area of shapes that have a perimeter of 5,6 , $7, \ldots$, units. <br> Ask learners to find out what shape quadrilateral has the largest area when the perimeter is, for example 24 cm . |

Syllabus ref.

## Suggested teaching activities

## C5.3 and

E5.3

Carry out calculations involving the circumference and area of a circle; Answers may be asked for in multiples of $\pi$.

Solve simple problems involving the arc length and sector area as fractions of the circumference and area of a circle; (for Core, where the sector angle is a factor of 360 ).

Carry out calculations involving the surface area and volume of a cuboid, prism and cylinder; answers may be asked for in multiples of $\pi$.

Carry out calculations involving the surface area and volume of a sphere, pyramid and cone; formulae will be given for the surface area and volume of the sphere, pyramid and cone in the question.

A useful starting point is revising how to calculate the circumference and area of a circle, using straightforward examples. Learners are expected to know the formulae.

Extend this by looking at how to find compound areas involving circles, for example, a circle with the radius of 5.3 cm is drawn touching the sides of a square. Ask learners "What area of the square is not covered by the circle?" the question can be extended to consider the area of waste material when cutting several circles of this size out of an A4 sheet of paper. (I)

The Khan academy website (www.khanacademy.org) includes a good explanation of arc length and sector area. Search 'Arc length from subtended angle' and 'Area of a sector'. The quiz for arc length includes challenge questions to check learners' understanding. (F)

The next step is to use examples to illustrate how to calculate the arc length and the sector area by using fractions of full circles. Learners will need to combine their work on sector area with area of a triangle work (syllabus reference 6.3) to find segment areas.

Starting with simple examples, draw the nets of a variety of solids asking learners if they are able to identify the solid from the net. It is useful for learners to understand that there are many different right and wrong ways to draw the net of a cube. Less able learners might appreciate the opportunity to work in groups to draw nets on card and to use these to make various geometrical shapes.

Next, demonstrate a purpose and use for drawing nets. For example, in the packaging industry there are many different interesting nets used to create boxes, particularly those that require little or no glue. An interesting homework activity would be to ask learners to collect lots of different packaging boxes to investigate the nets used to create them.

You can then ask learners to look at how to calculate the surface area of a cuboid and a cylinder, using the nets to help. Extend this to illustrating how to calculate the volume of a cuboid and a variety of prisms, including cylinders. Learners will find it useful to know the formula volume of prism $=$ cross-sectional area $\times$ length. A useful resource on this topic can be found on the Annenberg learner website (www.learner.org); 'Geometry 3D shapes $>$ surface area'.

Move on to using nets to illustrate how to calculate the surface area of a triangular prism, a pyramid and a cone. It will be useful for learners to understand how to obtain the formula $\pi r(r+s)$ for the surface area of a cone (where $s=$ slant length). You will also want to explain how to calculate the surface area of a sphere using the formula $4 \pi r^{2}$.

Use examples to illustrate how to calculate the volume of a pyramid (including a cone) using the formula $\frac{1}{3} \times$ area of base $\times$ perpendicular height. Also look at how to calculate the volume of a sphere using the formula $\frac{4}{3} \pi r^{3}$. Diagrams and formulae can be found at www.thoughtco.com; search for 'Math Formulas for Geometric Shapes'. Emphasise to learners

| Syllabus <br> ref. | Learning objectives |
| :--- | :--- |
|  |  |
| C5.5 | Carry out calculations involving the <br> areas and volumes of compound <br> shapes; answers may be asked for in <br> multiples of $\pi$ <br> challenge those with good memories to learn the given formulae too. |
| The final section is all about extending all the work from sections 4.1 to 4.4 to find the surface area and volume of a wide <br> variety of composite shapes. |  |
| Past and specimen papers |  |
| Past/specimen papers and mark schemes are available for the $\mathbf{0 5 8 0}$ syllabus to download at www.cambridgeinternational.org/support (F) |  |
| C5.1: Paper 23 Nov 2017 Q10 |  |
| C5.3: Paper 1 Nov 2016 Q22 |  |
| E5.3: Paper 42 June 17 Q5(b) and (c); Paper 22 Nov 2017 Q23 |  |
| C5.4: Paper 41 June 2017 Q5; Paper 42 Nov 2017 Q2 |  |
| C5.5: Paper 42, Nov 2017, Q2 |  |

## 6 Trigonometry

| Syllabus ref. | Learning objectives | Suggested teaching activities |
| :---: | :---: | :---: |
| C6.1 and E6.1 | Interpret and use three-figure bearings; measured clockwise from the North, i.e. $000^{\circ}-360^{\circ}$ | Introduce three-figure bearings and use examples of measuring and drawing involving bearings. You may want to link this work to that on scale drawings in topic 3.3. <br> Use examples to show how to calculate bearings, e.g. calculate the bearing of $B$ from $A$ if you know the bearing of $A$ from B. <br> Use a map to determine distance and direction (bearing) between two places, e.g. learners' home and school, etc. Maps from around the world can be found online at maps.google.com (I) |
| C6.2 and E6.2 | Apply Pythagoras' theorem and the sine, cosine and tangent ratios for acute angles to the calculation of a side or of an angle of a right-angled triangle. <br> Angles will be quoted in degrees. Answers should be written in degrees and decimals to one decimal place. | Revise squares and square roots. Use simple examples involving right-angled triangles to illustrate Pythagoras' theorem. Start with finding the length of the hypotenuse then move on to finding the length of one of the shorter sides. See 'Pythagoras' theorem' examples online on the Maths is fun website (www.mathsisfun.com). <br> This could be extended by exploring some of the 'Pythagoras proofs' on the Nrich website (https://nrich.maths.org). <br> Extend this work to cover diagrams where the right-angled triangle isn't explicitly drawn, or the problem is presented without a diagram, e.g. 'find the diagonal length across a rectangular field or the height of a building'. You may also want to use examples of triangles in different orientations and where the labels are different. For example, where the hypotenuse is labelled $a$ not $c$. This will check whether students really understand the theorem or whether they are just following a prescribed routine. (F) <br> When introducing trigonometry, spend some time on labelling the sides of triangles with a marked angle: adjacent, hypotenuse and opposite. Ask learners to work in groups to draw right-angle triangles with a $30^{\circ}$ angle of various sizes. Ask them to work out the ratio 'opposite side $\div$ adjacent side' for all the different triangles to find they should all be a similar value. <br> Then use examples involving the sine, cosine and tangent ratios to calculate the length of an unknown side of a rightangled triangle given an angle and the length of one side. Use a mix of examples, some examples where division is required and some examples where multiplication is required. For learners who struggle with rearranging the trigonometrical ratios it is possible to use the 'formula triangle approach'. <br> For more able learners, encourage the rearranging approach. Move on to examples involving inverse ratios to calculate an unknown angle given the length of two sides of a right-angled triangle. |

## Suggested teaching activities

|  | Solve trigonometric problems in two <br> dimensions involving angles of <br> elevation and depression. <br> Know that the perpendicular distance <br> from a point to a line is the shortest <br> distance to the line |
| :--- | :--- |
|  | Recognise, sketch and interpret graphs <br> of simple trigonometric functions. |
|  |  |
|  |  |

Solve a wide variety of problems in context using Pythagoras' theorem and trigonometric ratios (include work with any shape that may be partitioned into right-angled triangles). (I)

Use examples to illustrate how to solve problems involving bearings using trigonometry. (F)
For extended learners define angles of elevation and depression. Use examples to illustrate how to solve problems involving angles of elevation and depression using trigonometry.

Draw a sine curve and discuss its properties. Use the curve to show, for example, $\sin 150^{\circ}=\sin 30^{\circ}$. Repeat for the cosine curve.

You can build on the work learners did recognising and interpreting graphs of functions (E2.11). Learners need to know the values of $\sin (\theta), \cos (\theta)$ and $\tan (\theta)$ for $\theta=0,30,60,90$, and 180. The Khan Academy website (www.khanacademy.org) includes a useful proof 'Trig ratios of special triangles'.

Use the unit circle to help learners understand the relationship between different trigonometric equations, for example $\cos 30^{\circ}$ and $\cos 150^{\circ}$. The Maths is Fun web site (www.mathsisfun.com) demonstrates this well with a useful 'Unit circle' applet:
http://www.mathsisfun.com/geometry/unit-circle.html
http://www.mathsisfun.com/algebra/trig-interactive-unit-circle-flash.html
The Khan academy website (www.khanacademy.org/math/trigonometry/unit-circle-trig-func) also has some good material on the unit circle: 'The unit circle definition of sine, cosine, and tangent.' It uses radians consistently, so you may need to introduce radians to learners if you are going to use this site
'Tangled Trig Graphs' is a problem on the Nrich website (https://nrich.maths.org) that will be accessible to learners who have studied the unit circle and transformations of graphs (E2.11)

## Suggested teaching activities

| E6.4 | Solve problems using the sine and cosine rules for any triangle and the formula area of triangle $=\frac{1}{2} a b \sin C$, e.g. $\sin x=\frac{\sqrt{3}}{2}$ for values between $0^{\circ}$ and $360^{\circ}$; includes problems involving obtuse angles. | Rearrange the formula for the area of a triangle $\frac{1}{2} b h$ to the form $\frac{1}{2} a b \sin C$ (http://regentsprep.org has a useful resource to support this). Illustrate its use with a few simple examples. Explain that the letters in the formula may change from problem to problem, so learners should try to remember the pattern of two sides and the sine of the included angle. <br> Extend this to see if learners can use the formula to work out other problems, e.g. 'calculate the area of a segment of a circle given the radius and the sector angle' (using their knowledge of sector area work from topic 4.3) or 'calculate the area of a parallelogram given two adjacent side lengths and any angle'. (I) <br> Use examples to show how to solve problems using the sine rule, explaining that the version $\frac{a}{\sin A}=\frac{b}{\sin B}$ is preferable for finding a side and the version $\frac{\sin A}{a}=\frac{\sin B}{b}$ is preferable for finding an angle. <br> Use examples to show how to solve problems using the cosine rule. Make sure that learners either learn both rearrangements of the formula: <br> - to find a side $a^{2}=b^{2}+c^{2}-2 b c \cos A$ <br> - to find an angle $\cos A=\frac{b^{2}+c^{2}-a^{2}}{2 b c}$ <br> or can confidently rearrange from one to the other. <br> Give learners a set of questions where they can either use the sine rule or the cosine rule. Ask them not to work out the answers but instead to decide which rule to use. Explain how learners can tell whether they need the sine rule or the cosine rule, i.e. use the cosine rule when you know all three sides in a triangle or an enclosed angle and two sides, otherwise use the sine rule. <br> The Maths is Fun website (www.mathsisfun.com) has a useful page on 'Area of triangles without right angles'. |
| :---: | :---: | :---: |
| E6.5 | Solve simple trigonometrical problems in three dimensions including angle between a line and a plane. | Introduce problems in three dimensions by finding the length of the diagonal of a cuboid and determining the angle it makes with the base. Extend by using more complex figures, e.g. a pyramid. |

## Syllabus

ref.

Past and specimen papers
Past/specimen papers and mark schemes are available for the $\mathbf{0 5 8 0}$ syllabus to download at www.cambridgeinternational.org/support (F)
C6.1: Paper 41 June 2017 Q8
E6.2: Paper 43 June 2017 Q9 (a) and (b)
E6.3: Specimen Paper 4 Q8
E6.4: Paper 41 Nov 2017 Q10(a)
E6.5: Paper 21 June 2017 Q13

## 7 Vectors and transformations

| Syllabus ref. | Learning objectives | Suggested teaching activities |
| :---: | :---: | :---: |
| C7.1 and E7.1 | Describe a translation by using a vector represented by e.g. $\binom{x}{y}, \overrightarrow{A B}$, or $\mathbf{a}$. <br> Add and subtract vectors. <br> Multiply a vector by a scalar. | Use the concept of translation to explain a vector. Use simple diagrams to illustrate column vectors in two dimensions, explaining the significance of positive and negative numbers. <br> Introduce the various forms of vector notation. <br> Show how to add and subtract vectors algebraically by making use of a vector triangle. <br> Show how to multiply a column vector by a scalar and illustrate this with a diagram. <br> The 'vector journeys' problem on the Nrich website (https://nrich.maths.org) gives learners the opportunity to explore the use of vectors. There are also support and extension activities linked to this resource. |
| C7.2 and E7.2 | Reflect simple plane figures in horizontal or vertical lines. <br> Rotate simple plane figures about the origin, vertices or midpoints of edges of the figures, through multiples of $90^{\circ}$. <br> Construct given translations and enlargements of simple plane figures; positive and fractional scale factors for enlargements only; also includes negative scale factors for enlargements for extended learners. <br> Recognise and describe reflections, rotations, translations and enlargements; positive and fractional scale factors for enlargements only; also includes negative scale factors for enlargements for extended learners. | Draw an arrow shape on a squared grid. Use this to illustrate the following: reflection in a line (mirror line); rotation about any point (centre of rotation) through multiples of $90^{\circ}$ (in both clockwise and anti-clockwise directions); and translation by a vector. Several different examples of each transformation should be shown. Use the word image appropriately. <br> Investigate how transformations are used to make tessellations and produce an Escher-type drawing. For inspiration and step by step guides, see the website www.tessellations.org. (I) Ask learners to research the work of Maurits Cornelis Escher linked to tessellations; this can be set at homework. (I) <br> Use a pre-drawn shape on $(x, y)$ coordinate axes to complete several transformations using the equations of lines to represent mirror lines and coordinates to represent centres of rotation. Work with $(x, y)$ coordinate axes to show how to find: the equation of a simple mirror line given a shape and its (reflected) image; the centre and angle of rotation given a shape and its (rotated) image; and the vector of a translation. Emphasize all the detail that is required to describe each of the transformations. <br> Draw a triangle on a squared grid. Use this to illustrate enlargement by a positive integer scale factor about any point (centre of enlargement). Use both methods: counting squares and drawing rays. Show how to find the centre of enlargement given a shape and its (enlarged) image. |

Syllabus ref.

Learning objectives

## Suggested teaching activities

Revise the work from section 7.1. Use diagrams to help illustrate how to calculate the magnitude of a vector; link this to the work on Pythagoras' theorem from topic 6.2.

Explain the notation required, i.e. $\overrightarrow{A B}$ or a for vectors and for their magnitudes $|\overrightarrow{A B}|$ or $|\mathbf{a}|$ (with modulus signs).
Define a position vector and solve various problems in vector geometry. Explain to learners that in their answers to questions, they are expected to indicate $\mathbf{a}$ in some definite way, e.g. by an arrow or by underlining, thus $\overrightarrow{A B}$ or $\underline{a}$.

Past and specimen papers
Past/specimen papers and mark schemes are available for the $\mathbf{0 5 8 0}$ syllabus to download at www.cambridgeinternational.org/support ( $\mathbf{F}$ )

## C7.1: Paper 11 June 2017 Q13

C7.2: Paper 42 June 2013 Q2a (i) and (ii); Paper 42 June 2017 Q 2(a)
E7.3: Paper 21 June 2017 Q18

## 8 Probability

| Syllabus ref. | Learning objectives Suggested teaching activities |  |
| :---: | :---: | :---: |
| C8.1 and E8.1 | Calculate the probability of a single event as either a fraction, decimal or percentage. <br> Problems could be set involving extracting information from tables or graphs. | Use theoretical probability to predict the likelihood of a single event. For example, find the probability of choosing the letter M from the letters of the word mathematics. Use the formula: $\text { probability }=\frac{\text { favourable outcomes }}{\text { possible outcomes }}$ <br> Discuss when fractions, decimals or percentages are preferable for representing probabilities, e.g. if the probability is $\frac{2}{3}$ then a fraction is preferable because it is exact. <br> Learners use example questions that you've prepared or from textbooks. (I) |
| C8.2 and E8.2 | Understand and use the probability scale from 0 to 1 . | Discuss probabilities of 0 and 1 , leading to the outcome that a probability lies between these two values. Revise the language of probability associated with the probability scale. Use the probability scale by estimating frequencies of events occurring based on probabilities. <br> Ask learners to produce their own probability scale with events marked on it. Fix a string across the room. On one end, attach a card that says 'Certain' and on the other end attach a card that says 'Impossible'. In between, attach cards 'Fairly likely', 'Very likely', 'Not very likely', 'Equally likely' and others, if wanted. Pre-prepare cards that refer to each of the learners in the group and some to events that are either topical or of interest to the learners. It is useful if one card refers to an event that is almost certain and another to something that is almost impossible. Attach the topical event cards anywhere on the string between 'Certain' and 'Impossible'. Ask learners to discuss among themselves the order in which they should appear. Next, label the 'Impossible' card as 'Probability 0 ' and the 'Certain' card as 'Probability 1 ' and explain that probabilities are measured between 0 and 1 . Ask for suggestions for numerical values (in decimals or fractions) for the topical card events. Write these values on blank cards and attach them above the event cards. <br> Ask learners to find out the meaning of mutually exclusive and exhaustive. (I) |
| C8.3 | Understand that the probability of an event occurring $=1-$ the probability of the event not occurring. | Use examples to show that the 'probability of an event occurring $=1-$ the probability of the event not occurring', including those where there are only two outcomes and those when there are more than two outcomes. |


| Syllabus ref. | Learning objectives Suggested teaching activities |  |
| :---: | :---: | :---: |
| C8.4 and E8.4 | Understand relative frequency as an estimate of probability. <br> Expected frequency of occurrences. | Compare estimated experimental probabilities, or relative frequency, with theoretical probabilities. Learners need to recognise that when experiments are repeated different outcomes may result, and increasing the number of times an experiment is repeated generally leads to better estimates of probability. <br> Conduct a class experiment into rolling dice 300 times, e.g. 15 pairs of learners rolling a dice 20 times each. Collect and combine results from groups to create a large sample set, show how estimates change as more data is added to the set. <br> Repeat the experiment where the theoretical probability is not known, e.g. the chance of a drawing pin landing point down when thrown in the air. Try 'Buffon's Needle' activity on the Maths is fun website (hwww.mathsisfun.com). <br> Carry out experiments to sample the number of unknown coloured counters in a bag. Ask learners to suggest how many of each type of coloured counter there are in the bag, given the known total. |
| C8.5 | Calculate the probability of simple combined events, using possibility diagrams, tree diagrams and Venn diagrams. <br> In possibility diagrams, outcomes will be represented by points on a grid, and in tree diagrams, outcomes will be written at the end of branches and probabilities by the side of the branches. <br> For Core, Venn diagrams will be limited to two sets. | Roll two different dice, or spin two spinners, and list all the outcomes. Use simple examples to illustrate how possibility diagrams and tree diagrams can help to organise data. <br> Use possibility diagrams and tree diagrams to help calculate probabilities of simple combined events, paying close attention to how diagrams are labelled. <br> The article 'Probability calculations from tree diagrams' on the Nrich website (https://nrich.maths.org/9648) suggests a set of activities that introduce students to combined events in an intuitive way using tree diagrams as a means of recording and visualising the outcomes of combined events. The examples are set at different levels of complexity. <br> The article 'Tree diagrams, 2-way Tables and Venn Diagrams' also on the Nrich website (https://nrich.maths.org/9861) considers a range of diagrammatic representations for probability. The resources include some detailed examples of how different representations could be used to support the solution to example problems. These could be used to stimulate discussion with learners. <br> Resource Plus <br> Skills Pack: Probability of combined events <br> The Skills Pack includes lessons on: <br> - possibility space diagrams <br> - area and volume <br> - drawing and interpreting tree diagrams <br> - tree diagrams \& more complex probabilities. |

Syllabus ref.

E8.6

Calculate conditional probability using
Venn diagrams, tree diagrams and tables.

For example, two dice are rolled. Given that the total showing on the two dice is 7 , find the probability that one of the dice shows the number 2 .

Try the 'In a box' probability problem on the Nrich website (https://nrich.maths.org/919). (I)

## Resource Plus

## Skills Pack: Probability of combined events

The Skills Pack includes a lesson on conditional probability, and tree diagrams \& more complex probabilities $\qquad$

## Suggested teaching activities

## Past and specimen papers

Past/specimen papers and mark schemes are available for the $\mathbf{0 5 8 0}$ syllabus to download at www.cambridgeinternational.org/support (F)
C8.1: Paper 32 June 2017 Q3(a)
C8.3: Paper 12 June 2017 Q2; Paper 33 June 2017 Q7 (d(i))
C8.4: Paper 33 June 2017 Q7(d(ii)); Paper 43 June 2017 Q5 (c)
C8.5: Specimen Paper 2 Q22; Paper 23 June 2017 Q6
E8.6: Specimen Paper 4 Q2(b(iii)); Paper 42 June 2017 Q6

## 9 Statistics

| Syllabus ref. | Learning objectives Suggested teaching activities |  |
| :---: | :---: | :---: |
| C9.1 and E9.1 | Collect, classify and tabulate statistical data. | Use simple examples to revise collecting data and presenting it in a frequency (tally) chart. For example, record the different makes of car in a car park, or record the number of words on the first page of a series of different books. <br> Ask learners to conduct an experiment of this type, tabulating their data. <br> Use examples to classify data using statistical terminology, e.g. discrete, continuous, numerical (quantitative), nonnumerical (qualitative). Use examples to show how to draw simple inferences from statistical diagrams, and tables including two-way tables. |
| C9.2 and E9.2 | Read, interpret and draw simple inferences from tables and statistical diagrams. <br> Compare sets of data using tables, graphs and statistical measures. <br> Appreciate restrictions on drawing conclusions from given data. | Explore the Gapminder website (https://www.gapminder.org/) for some interesting ways of displaying data and access to a wide range of data sets. <br> Learners can be encouraged to bring in examples of data used in the media. Discuss how the data is represented. Is it designed to be informative, or to deliver a particular message, or to sell something? <br> 'Interpreting Bar Charts, Pie Charts, Box and Whisker Plots S5' on the STEM learning website (www.stem.org.uk) gives learners the opportunity to interpret bar charts and pie charts, and helps them appreciate the benefits and limitations of these representations. The second part of the resource also compares box-and-whisker plots, which his useful for topic E9.6. |
| C9.3 and E9.3 | Construct and interpret bar charts, pie charts, pictograms, stem-and-leaf diagrams, simple frequency distributions, histograms with equal intervals and scatter diagrams. | Use the data collected in the activity recommend for topic 9.1 to construct a pictogram, a bar chart and a pie chart. Point out that the bars in a bar chart can be drawn apart. <br> Use an example to show how discrete data can be grouped into equal classes. Draw a histogram to illustrate the data (i.e. with a continuous scale along the horizontal axis). Point out that this information could also be displayed in a bar chart (i.e. with bars separated) because data is discrete. <br> Investigate the length of words used in two different newspapers and present the findings using statistical diagrams (links to newspapers can be found online at http://onlinenewspapers.com). <br> Record sets of continuous data, e.g. heights, masses, etc., in grouped frequency tables. Use examples that illustrate equal class widths for core learners and unequal class widths for extended learners. Draw the corresponding histograms. <br> Emphasize the fact that for continuous data bars of a histogram must touch. |


| Syllabus <br> ref. | Learning objectives |  |
| :--- | :--- | :--- |
|  | Extended learners also cover <br> histograms with unequal intervals; for <br> unequal intervals on histograms, areas <br> are proportional to frequencies and the <br> vertical axis is labelled 'frequency <br> density'. | On the Mr Barton Maths website eBook 'The Maths E-Book of Notes and Examples' (http://mrbartonmaths.com), there is <br> a section on bar charts and histograms. Use this section to illustrate to extended learners why frequency density is a fairer <br> way to represent data than frequency on the vertical axis. Label the vertical axis of a histogram as 'frequency density' and <br> show that the area of each bar is proportional to the frequency. Show how to calculate frequency densities from a <br> frequency table with grouped data and how to calculate frequencies from a given histogram. <br> Explain how to draw scatter diagrams with simple examples (you may choose to do this at the same time as topic 9.6). <br> Explore the Gapminder website (https://www.gapminder.org/) for innovative approaches to scatter diagrams and videos <br> that you could use to engage learners using some real life topical contexts. |
| C9.4 and <br> E9.4 | Calculate the mean, median, mode and <br> range for individual and discrete data <br> and distinguish between the purposes <br> for which they are used. | Show how to work out the mean, the median and the mode from a list of data or from a frequency table. <br> Explain that if there are two middle values, they need to find the half-way point for the median, because there can only be <br> one median but that there can be more than one mode or no mode. |
| Use simple examples to highlight how these averages may be used. For example, in a discussion about average salaries the |  |  |
| owner of a company with a few highly paid managers and a large work force may wish to quote the mean wage rather than |  |  |
| the median. 'Mean, median and mode' on the mathsteacher.com website (www.mathsteacher.com) has examples of when |  |  |
| to use the different averages for different situations. |  |  |
| Use the 'Life Expectancy' PowerPoint and resources on the Gapminder website (https://www.gapminder.org/) to explore |  |  |
| averages in a topical real-life situation. |  |  |
| Include examples where the mean is given and the number of people, total or an individual value needs to be found. |  |  |


| Syllabus ref. | Learning objectives Suggested teaching activities |  |
| :---: | :---: | :---: |
| E9.5 | Calculate an estimate of the mean for grouped and continuous data. <br> Identify the modal class from a grouped frequency distribution. | Use examples to show how to calculate an estimate for the mean of data in a grouped frequency table using the midinterval values. Explain how the modal class can be found in a grouped frequency distribution. <br> Look at the examples and questions on the 'Centre for innovation in mathematics teaching' website (http://www.cimt.org.uk/). (I) <br> The Maths is Fun website (www.mathsisfun.com) has a good explanation of how to estimate the mean for grouped data, including identifying the modal class. See 'Mean, Median and Mode from Grouped Frequencies'. <br> Extension activity: Explain how to find the interval that contains the median; for more able learners you could show them the idea of linear interpolation. |
| E9.6 | Construct and use cumulative frequency diagrams. <br> Estimate and interpret the median, percentiles, quartiles and inter-quartile range. <br> Construct and interpret box-andwhisker plots. | Explain cumulative frequency and use an example to illustrate how a cumulative frequency table is constructed. Draw the corresponding cumulative frequency curve emphasising that points are plotted at upper class limits; the curve must always be increasing; and highlight its distinctive shape. Explain that this can be approximated by a cumulative frequency polygon. <br> Use a cumulative frequency curve to help explain and interpret percentiles. Introduce the names given to the 25th, 50th and 75th percentiles and show how to estimate these from a graph. Show how to estimate the inter-quartile range from a cumulative frequency diagram. Explain how to use a cumulative frequency curve to complete a frequency table. <br> 'Interpreting frequency graphs, cumulative frequency graphs, box and whisker plots S6' on the Stem learning website (www.stem.org.uk) gives learners the opportunity to interpret simple frequency distributions and cumulative frequency diagrams, including using box-and-whisker diagrams, to display the median, percentiles, quartiles and inter-quartile range. <br> The second part of the 'Interpreting Bar Charts, Pie Charts, Box and Whisker Plots S5' on the STEM learning website (www.stem.org.uk) compares box-and-whisker plots. <br> Use specimen Paper 2, Q24 (reference also to E1.16 Personal and household finances). (I) (F) |


| Syllabus ref. | Learning objectives | Suggested teaching activities |
| :---: | :---: | :---: |
| C9.7 and E9.7 | Understand what is meant by positive, negative and zero correlation with reference to a scatter diagram. | Use simple examples of scatter diagrams to explain the terms and meanings of positive, negative and zero correlation. Revise drawing scatter diagrams and describe the resulting correlation. Explain why and where scatter graphs are useful, e.g. in making predictions. <br> Ask learners to collect some bivariate data of their choice and to predict the correlation, if any, that they expect to find. For example, height and arms span for members of the class. Use collected data to draw a scatter diagram and to then look for the expected correlation. Discuss the results. <br> You could use the 'How does income relate to life expectancy' presentation on the Gapminder website (https://www.gapminder.org/) to demonstrate the use of correlation to explore social problems. <br> Explain that if there are too few points on a scatter diagram a correlation may appear apparent when in fact there is no real relationship between the variables. Learners should understand that a correlation does not prove cause and effect it just provides evidence to support a potential relationship and/or identify an area for further research. For example, it may be that a third unidentified variable is causing the apparent correlation. Learners could do a web search for the 'Televisions, Physicians, and Life Expectancy' problem to demonstrate this phenomenon. (I) <br> The 'David and Goliath' problem on the Nrich website (https://nrich.maths.org/7360) is a nice activity. (I) |
| C9.8 and E9.8 | Draw, interpret and use lines of best fit by eye. | Explain, with diagrams, that the purpose of a good line of best fit is to have the sum of the vertical distances from each point to the line as small as possible. In simpler terms, ask learners to aim for a similar number of points on each side of the line and as many points as possible on the line or as close to it as possible. <br> Draw diagrams showing bad lines of best fit explaining what is wrong with them. For example, a common error made by learners is to draw the line of best fit through the origin when that doesn't fit with the trend of the data. |
| Past and specimen papers |  |  |
| Past/specin <br> C9.1: Pape <br> C9.3: Pape <br> E9.3: Pape <br> E9.5: Pape <br> E 9.6: Spe <br> C9.7: Pape <br> C9.8: Pape | papers and mark schemes are available <br> 13 June 2017 Q6; Paper 32 June 2017 Q3 <br> 31 June 2017 Q3(a(i,ii,iii)); Specimen Pap <br> 22 June 2017 Q3; Paper 43 June 2017 Q 5 <br> 41 June 2017 Q2(c) <br> men Paper 2 Q24 (reference also to E1.16 <br> 11 June 2017 Q22 (a) (b) <br> 11 June 2017 Q22 9(c) | or the $\mathbf{0 5 8 0}$ syllabus to download at www.cambridgeinternational.org/support ( $\mathbf{F}$ ) r 1 Q3 <br> Personal and household finances) |

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